A Cognitive Component Analysis of Word Problem Solving

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Abstract

Word problem solving processes generally consist of four cognitive components, translation, integration, planning, and execution (Mayer, 1985, 1987; Mayer, Tajika, & Stanley, 1991). The problem comprehension process includes both translation and integration components and the problem solving process includes both planning and execution components. In the translation component, the student must translate each proposition from the problem into an internal representation. In the integration component, the student must select and combine information into a coherent representation of the entire problem. In the planning component, the student must break the problem down into a series of steps and generating a plan for solving it. In the execution component, the student must carry out mathematical operations. Problems were developed to examine each of these four components. Two experiments were carried out to explore which cognitive components influence difficulty in solving arithmetic word problems. In the first experiment, one hundred and ten fifth-graders were tested on these four problems. Subjects were divided into two groups, high problem solvers and low problem solvers, based on tests involving three cognitive components. The results showed that there was an interaction between component and condition, in which the difference in scores between high problem solvers and low problem solvers was the largest on the problem integration component. In the second experiment, one hundred and seventy-four fifth-graders were tested on the problem comprehension process. The results showed that subjects performed best when they were presented relational pictures integrating each sentence of a word problem. The results of these two experiments are discussed in terms of the problem solving components.

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The process of word problem solving generally can be broken down into two main processes, problem comprehension and problem solution. In problem comprehension, a student forms an internal representation from the words and sentences of a word problem. In problem solution, the student applies appropriate arithmetic operators to the internal representation to get an answer. Mayer (1985, 1987, 1992; Mayer, Tajika, & Stanley, 1991) has analyzed these cognitive processes further: translation, integration, planning and monitoring (which are referred to here as planning), and execution (see Figure 1). Translation and integration are components in problem comprehension and planning and execution are components in problem solution.

In the translation component, the student must translate each proposition from the problem into an internal representation. In the integration component, the student must select and combine information into a coherent representation of the entire problem. In the planning component, the student must break the problem down into a series of steps and generate a plan for solving it. In the execution component, the student must carry out mathematical operations. The first three components are different from those involving the execution component, because the latter requires procedural knowledge for carrying out arithmetic computations only, whereas the former require a variety of declarative knowledge.

We conducted two experiments to examine which cognitive components is or are closely related to solution accuracy.

In the first experiment, we explored which of the above three cognitive components differentiate high and low problem solvers among students who achieve low scores and students who achieve high scores in computation, based on their scores on the execution problems (Ishida & Tajika, 1993). Problems were developed to test each of these four components (see Mayer, Tajika, & Stanley, 1991). The test consisted of (1) six items on the translation problem, (2) six items on the integration problem, (3) six items on the planning problem, and (4) fifteen items on the execution problem. Examples of the four problem types and their possible answers are given below. These problems were used in the experiment of Mayer, Tajika, & Stanley (1991).

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**Figure 1.** Cognitive components in word problem solving.
(1) A problem evaluating the translation component is: "Which number sentence is correct?" Taro has 5 more marbles than Hanako. (a) Taro's marbles = 5 + Hanako's marbles, (b) Taro's marbles + 5 = Hanako's marbles, (c) Taro's marbles + Hanako's marbles = 5, or (d) Taro's marbles = 5.

(2) A problem evaluating the integration component is: "Which numbers are needed to solve this problem?" Yoshio had 500 yen for lunch. He bought a sandwich for 250 yen, an apple for 70 yen, and a milk for 80 yen. How much money did he spend? (a) 500, 250, 70, 80, (b) 250, 70, 80, (c) 250, 80, or (d) 500.

(3) A problem evaluating the planning component is: "Which operations should you carry out to solve this problem?" If it costs 500 yen per hour to rent roller skates, what is the cost of using the skates from 1:00 p.m. to 3:00 p.m.? (a) subtract, then multiply, (b) subtract, then divide, (c) add, then divide, or (d) multiply only.

(4) A problem evaluating the execution component is: "Solve this problem." \(42 \div 6 = \). (a) 6, (b) 7, (c) 8, (d) none of these.

The labels (a), (b), (c), and (d) correspond to four possible answers which subjects can choose for each problem. The correct answers are (a), (b), (a), and (b), respectively.

One hundred and ten fifth-grade students participated in this experiment. They were divided into two groups, high and low computational skill groups, based on their scores on the execution problems. Within each group, the students were further divided into two groups, high and low problem solving ability groups (high problem solvers and low problem solvers), based on the total scores of the remaining three problems (e.g., translation, integration, and planning problems).

Table 1 and Table 2 show the results of the first experiment. The results showed significant interactions of component (translation vs. integration vs. planning) and condition (high problem solvers vs. low problem solvers) in both computational skill groups \(F(2,56) = 3.45\), under the high computational skill group and \(F(2,44) = 4.41\), under the low computational skill group, both \(p < .05\). That is, the difference in

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Mean proportion correct (and standard deviations) in each condition under the high computational skill group</th>
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<tbody>
<tr>
<td></td>
<td>Component</td>
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<tr>
<td></td>
<td>Translation</td>
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<tr>
<td>Group</td>
<td>Total</td>
</tr>
<tr>
<td>High-</td>
<td>.90</td>
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<tr>
<td>solver</td>
<td>(.07)</td>
</tr>
<tr>
<td>Low-</td>
<td>.51</td>
</tr>
<tr>
<td>solver</td>
<td>(.11)</td>
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</table>

Note. Fifteen students were selected as high problem solvers and fifteen students were as low problem solvers under the high computational skill group.
scores between high problem solvers and low problem solvers was the largest on the integration problem in each computational skill group.

We interpreted the results of the first experiment in terms of the problem solving components. When arithmetic word problem solving is broken down into four components, namely translation, integration, planning, and execution components, the integration component, which selects and combines relevant information into a coherent representation of the problem, seems to be a more critical factor in terms of solution accuracy. The difference in performance between high problem solvers and low problem solvers was pronounced in problems evaluating the integration component.

In the second experiment, we attempted to examine which of the two cognitive components, translation and integration components, is more closely associated with solution accuracy in arithmetic word problems. Problems designed to measure the translation and integration components in this experiment were different from those in the first experiment. Three problems evaluating the translation component asked students to identify the contents of each statement of each word problem. Problems for the integration component consisted of two types of questions: diagrams and relational pictures. Students were asked to fill in appropriate numbers in each of diagrams or of relational pictures in the space provided (see Figure 2–a to Figure 3–b).

Subjects were one hundred and seventy-four fifth-grade students who were divided into six groups. Two independent variables (between-subjects variables) were experimental condition and problem type. There were three levels of experimental condition; relational picture group, diagram group, and control group. There were two levels of problem type; easy problem and hard problem.

An easy problem was different from a hard one in the number of steps to solve it. Each easy problem only requires one step to solve the problem, whereas a hard problem requires two or more steps to solve it. An example of an easy problem is as follows: “There are forty students in Masao’s class. The number of boys is 0.6 times as many as the total number of the students in his class. How many boys are there in

<table>
<thead>
<tr>
<th>Group</th>
<th>Total</th>
<th>Translation</th>
<th>Integration</th>
<th>Planning</th>
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</thead>
<tbody>
<tr>
<td>High-solver</td>
<td>.74</td>
<td>.68</td>
<td>.89</td>
<td>.65</td>
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<tr>
<td>( .11 )</td>
<td></td>
<td>( .22 )</td>
<td>( .12 )</td>
<td>( .19 )</td>
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<tr>
<td>Low-solver</td>
<td>.41</td>
<td>.42</td>
<td>.36</td>
<td>.45</td>
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<tr>
<td>( .13 )</td>
<td></td>
<td>( .16 )</td>
<td>( .28 )</td>
<td>( .14 )</td>
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</table>

Note. Twelve students were selected as high problem solvers and twelve students were as low problem solvers under the low computational skill group.
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An example of a hard problem is as follows: “There are forty students in Masao’s class. The number of boys is 0.6 times as many as the total number of the students in his class. How many girls are there in his class?”

Prior to solving each of eight word problems, all subjects were asked a question about each statement of the word problems. An example of the questions for a word problem is as follows: “How many students are there in Masao’s class? What is the ratio of the number of boys in his class when the ratio of the total number of the students in his class is 1? What are you asked on this word problem?” The maximum score of the translation problem was twenty-four. Both subjects of the diagram group and the relational picture group were also asked to fill in appropriate numbers in the diagram in the space provided. The maximum score of the diagram problem was twenty-four. In addition, subjects in the relational picture group were asked to fill in appropriate numbers in the relational picture in the space provided. The maximum score of the relational picture problem was sixteen.

After subjects finished answering each of the eight questions corresponding to their experimental conditions, they solved each word problem.

The results of the second experiment are shown in Table 3. There were no differences in the results of the pretest among six experimental condition groups. Three experimental condition groups also showed no difference in the scores of problems evaluating the translation component. However, on the final test of the word problem solving, we found significant main effects for experimental condition and of problem type variables ($F(2,168) = 4.48$, $p < .05$ and $F(1,168) = 42.31$, $p < .01$, respective-
Subjects in the relational picture group performed best. As expected, students solved easy problems more accurately than hard problems.

We interpreted the results of the second experiment in terms of the problem solving components, as in the first experiment. The results indicated that the students who solved problems involving relational pictures, that is, problems emphasizing the integration component, performed best on arithmetic word problem solving. When problem comprehension process of arithmetic word problem solving can be broken down into translation and integration components, it appears that the integration component influences difficulty levels of arithmetic word problems.

The results of both experiments suggest the importance of the integration component for more accurate solutions of arithmetic word problems over the other components.

References


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