Maximal $CP$ violation in the Higgs Sector and its Effect on the $\rho$ Parameter

Girish C. JOSHI\textsuperscript{(a)}\footnote{E-mail:joshi@bradman.phys.unimelb.ac.au} Masahisa MATSUDA\textsuperscript{(b)}\footnote{E-mail:masa@auephyas.aichi-edu.ac.jp} and

Morimitsu TANIMOTO\textsuperscript{(c)}\footnote{Permanent address:Science Education Laboratory, Ehime University, 790 Matsuyama, JAPAN}

\textsuperscript{(a)} Department of Physics, University of Melbourne

\textit{Parkville, Victoria 3052, AUSTRALIA}

\textsuperscript{(b)} Department of Physics and Astronomy, Aichi University of Education

\textit{Kariya, Aichi 448, JAPAN}

\textsuperscript{(c)} Institut für Theoretische Physik, Universität Wien

\textit{Boltzmanngasse 5, A-1090 Wien, AUSTRIA}

ABSTRACT

We study the conditions of maximal $CP$ violation in the neutral Higgs mass matrix of the two Higgs doublet model. We get fixed values of $\tan \beta$ and constraints on the Higgs potential parameters. Two neutral Higgs scalars are constrained to be lighter than the charged Higgs scalar and these two Higgs scalars are expected to be almost degenerate due to the smallness of the $h$ parameter, where $h$ is the $CP$ violating coupling constant of the Higgs interaction. The radiative correction of the $\rho$ parameter from the Higgs scalar exchange is rather small and its sign negative for a wide range of Higgs masses. It follows that maximum $CP$ violation in the two Higgs doublet model is safely allowed for the $\rho$ parameter without the custodial symmetry.
The physics of $CP$ violation has attracted much recent attention in the light that the $B$-factory will go on line in the near future. In the standard model (SM), the origin of such $CP$ violation is reduced to the phase in the Kobayashi-Maskawa matrix[1]. However, there has been a general interest in considering other approaches to $CP$ violation since many alternate sources exist.

In this paper, we consider the simplest and most attractive extension of the standard Higgs sector, namely the type II two Higgs doublet model (THDM)[2], yielding both charged and neutral Higgs bosons as physical states. The THDM with the soft breaking term of the discrete symmetry demonstrates explicit or spontaneous $CP$ violation[3]. Some authors have proposed to search for the $CP$-violating observables in the Higgs sectors, where physical effects occur in the electric dipole moment of the neutron[4] and electron[5], or as asymmetries in the top quark production or decay[6]. Such loop contributions are generally small and decrease with increasing Higgs masses. Direct $CP$-violating Higgs productions were also predicted in $e^+e^-$ colliders[7,8]. On the other hand, the radiative corrections arising from the Higgs scalar contribution to the vector-boson self-energyes have been studied in this model[9]. In this context, Pomarol and Vega proposed a new way to constrain $CP$ violation in the Higgs sector using the experimental value of the $\rho$ parameter[10]. Since a custodial $SU(2)$ symmetry cannot be defined in the $CP$-violating Higgs potential, the radiative corrections to $\rho$ are unavoidable.

In this paper, we study the neutral Higgs mass matrix with maximal $CP$ violation and its effect on the $\rho$ parameter in the THDM[7,10]. We have found that maximal $CP$ violation is realized under the fixed values of $\tan \beta$ with two constraints of parameters in the Higgs potential. Taking these conditions into account, we investigate the
contribution of the Higgs sector to the $\rho$ parameter.

First, we will discuss maximal $CP$ violation in the THDM. The Higgs potential with $CP$ violating terms in the THDM can be written as:

$$V_{\text{Higgs}} = \frac{1}{2} g_1 (\Phi_1^\dagger \Phi_1 - |v_1|^2)^2 + \frac{1}{2} g_2 (\Phi_2^\dagger \Phi_2 - |v_2|^2)^2$$

$$+ g (\Phi_1^\dagger \Phi_1 - |v_1|^2)(\Phi_2^\dagger \Phi_2 - |v_2|^2)$$

$$+ g' (\Phi_1^\dagger \Phi_2 - v_1^* v_2)^2 + \text{Re} [h (\Phi_1^\dagger \Phi_2 - v_1^* v_2)^2]$$

$$+ \xi \left( \frac{\Phi_1}{v_1} - \frac{\Phi_2}{v_2} \right)^\dagger \left( \frac{\Phi_1}{v_1} - \frac{\Phi_2}{v_2} \right),$$

(1)

where $\Phi_1$ and $\Phi_2$ couple with the down-quark and the up-quark sectors respectively and the vacuum expectation values are defined as $v_1 \equiv <\Phi_1^0>_{\text{vac}}$ and $v_2 \equiv <\Phi_2^0>_{\text{vac}}$. We do not concern ourselves here with a specific model of $CP$ violation, but instead consider a general parametrization using the notation developed by Weinberg[11]. We take $h$ to be real and set

$$v_1^* v_2 = |v_1 v_2| \exp(i\phi),$$

(2)

as a phase conversion. We define the neutral components of the two Higgs doublets using three real fields $\phi_1, \phi_2, \phi_3$ and the Goldstone boson $\chi^0$ as follows:

$$\Phi_1^0 = \frac{1}{\sqrt{2}} \{ \phi_1 + \sqrt{2} v_1 + i(\cos \beta \chi^0 - \sin \beta \phi_3) \},$$

$$\Phi_2^0 = \frac{1}{\sqrt{2}} \{ \phi_2 + \sqrt{2} v_2 + i(\sin \beta \chi^0 + \cos \beta \phi_3) \},$$

(3)

where $\tan \beta \equiv v_2/v_1$. The real fields $\phi_1$ and $\phi_2$ are scalar particles while $\phi_3$ is pseudo-scalar in the limit of $CP$ conservation. $CP$ violation will occur via the scalar-pseudoscalar interference terms in the neutral Higgs mass matrix. Maximal $CP$ violation was defined on a new basis by Georgi[12], where now the Goldstone boson decouples from the $\Phi_2$ doublet. The neutral Higgs scalars $H^0, H^1, H^2$ on this new basis
are given by the following rotation;
\[
\begin{pmatrix}
  H^0 \\
  H^1 \\
  H^2
\end{pmatrix} = \begin{pmatrix}
  \cos \beta & \sin \beta & 0 \\
  -\sin \beta & \cos \beta & 0 \\
  0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
  \phi_1 \\
  \phi_2 \\
  \phi_3
\end{pmatrix} .
\] (4)

Denoting the orthogonal matrix \( O \) that relates this basis with the mass eigenstates (physical states) \( \varphi_1, \varphi_2 \) and \( \varphi_3 \) as
\[
\begin{pmatrix}
  H^0 \\
  H^1 \\
  H^2
\end{pmatrix} = O \begin{pmatrix}
  \varphi_1 \\
  \varphi_2 \\
  \varphi_3
\end{pmatrix} ,
\] (5)
maximal \( CP \) violation is defined when
\[
O_{i1}^2 = O_{i2}^2 = O_{i3}^2 = \frac{1}{3} ,
\] (6)
which was discussed by Méndez and Pomarol[7].

In order to get maximal CP violation we investigate the eigenvectors and eigenvalues, which are given by solving the \( 3 \times 3 \) neutral Higgs mass matrix. The Higgs mass matrix elements of the neutral Higgs mass matrix \( M^2 \) in the \( \phi_1, \phi_2 \) and \( \phi_3 \) basis of eq.(3) are given by
\[
\begin{align*}
M_{11}^2 &= 2g_1|v_1|^2 + g'|v_2|^2 + \frac{\xi + \text{Re}(hv_1^* v_2^2)}{|v_1|^2} , \\
M_{22}^2 &= 2g_2|v_2|^2 + g'|v_1|^2 + \frac{\xi + \text{Re}(hv_1^* v_2^2)}{|v_2|^2} , \\
M_{33}^2 &= (|v_1|^2 + |v_2|^2) \left[ g' + \frac{\xi - \text{Re}(hv_1^* v_2^2)}{|v_1|v_2|^2} \right] , \\
M_{12}^2 &= |v_1v_2|(2g + g') + \frac{\text{Re}(hv_1^* v_2^2) - \xi}{|v_1v_2|} , \\
M_{13}^2 &= -\sqrt{|v_1|^2 + |v_2|^2} \text{Im}(hv_1^* v_2^2) , \\
M_{23}^2 &= -\sqrt{|v_1|^2 + |v_2|^2} \text{Im}(hv_1^* v_2^2) ,
\end{align*}
\] (7)
which is the symmetric mass matrix. Denoting the orthogonal matrix \( U \) to diagonalize

\[
\begin{pmatrix}
  H^0 \\
  H^1 \\
  H^2
\end{pmatrix} = U \begin{pmatrix}
  \varphi_1 \\
  \varphi_2 \\
  \varphi_3
\end{pmatrix} .
\] (8)
this mass matrix, $O$, which is defined in eq.(5), is obtained by

$$O = \begin{pmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} U . \quad (8)$$

We then have

$$O_{11} = \cos \beta U_{11} + \sin \beta U_{21} ,$$
$$O_{12} = \cos \beta U_{12} + \sin \beta U_{22} ,$$
$$O_{13} = \cos \beta U_{13} + \sin \beta U_{23} . \quad (9)$$

Let us consider the condition for maximal $CP$ violation in the neutral Higgs sector. The trivial case that $O_{11} = O_{12} = O_{13}$ is realized by $U_{i1} = U_{i2} = U_{i3}(i = 1, 2)$. However, these relations for the matrix $U$ are forbidden by unitarity. So, we consider instead the case that two matrix elements among $O_{i1}(i = 1, 2, 3)$ are equal without fixing the value of $\tan \beta$. Then the value of $\tan \beta$ can be tuned so as to give three equal matrix elements. First, we study the case of $O_{12} = O_{13}$. This relation is obtained if

$$U_{12} = U_{13} , \quad U_{22} = U_{23} \quad (10)$$

or

$$\tan \beta = \frac{U_{12} - U_{13}}{U_{23} - U_{22}} . \quad (11)$$

In the case of eq.(10), the orthogonal matrix $U$ is specified as follows:

$$U = \begin{pmatrix} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \sin \theta \\ -\sin \theta & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \cos \theta \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \quad (12)$$

where $\theta$ is an arbitrary rotation angle. It is easy to show that this orthogonal matrix is obtained when both $M_{12}^2 = 0$ and $M_{13}^2 = 0$ are derived after rotating $M^2$ on the (1-2)
plane with the angle $\theta$. The maximal mixing is then given on the new (2-3) plane. We can easily find this case in the Higgs mass matrix of eq.(7).

In the case of eq.(11), the situation is somewhat complicated. Let us denote by $V$ the orthogonal matrix required to diagonalize the Higgs mass matrix on the new basis after rotating $M^2$ on the (1-2) plane with the angle $-\beta$. We then have

$$U_{12} - U_{13} = \cos \beta (V_{12} - V_{13}) - \sin \beta (V_{22} - V_{23}),$$

$$U_{23} - U_{22} = \cos \beta (V_{23} - V_{22}) + \sin \beta (V_{13} - V_{12}).$$  \hspace{1cm} (13)

If $V_{12} = V_{13}$ is satisfied, with $V_{22} \neq V_{23}$, the relation of eq.(11) is reproduced. In general, this condition could exist for an orthogonal matrix. However, we cannot get this solution without fixing the value of $\tan \beta$ in the mass matrix in eq.(7).

Studying the cases $O_{11} = O_{13}$ and $O_{11} = O_{12}$ does not add new conditions to our result, because the exchange of the rotation axis gives the same conditions as in the above case. Thus, we consider only the case in eq.(10)(or (12)) in order to get maximal $CP$ violation.

Let us search for the constraints on the parameters in the Higgs potential to give the orthogonal matrix $U$ in eq.(12). These are easily found by rotating the Higgs mass matrix $M^2$ with $\theta = \beta$ so as to make the (1,3)(and then (3,1)) component zero. The orthogonal matrix $U_0$ is given by

$$U_0 = \begin{pmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \hspace{1cm} (14)$$

The transformed matrix $M'^2 = U_0^* M^2 U_0$ is then given by

$$M'_{11} = 2g_1 \cos^4 \beta + 2g_2 \sin^4 \beta + 4(\xi - g) \sin^2 \beta \cos^2 \beta,$$

$$M'_{22} = 2(g_1 + g_2 + 2g - 2\xi) \sin^2 \beta \cos^2 \beta + g' + \xi + h \cos 2\phi,$$
\[ M'_{33} = g' + \xi - h \cos 2\phi , \]
\[ M'_{12} = \sin \beta \cos \beta \left[ \cos 2\beta (g_1 + g_2 + 2g - 2\xi) + g_1 - g_2 \right], \quad (15) \]
\[ M'_{13} = 0 , \]
\[ M'_{23} = -h \sin 2\phi , \]

in \( v^2 \equiv |v_1|^2 + |v_2|^2 \) units and the parameter \( \xi \) is defined as \( \xi = \xi / |v_1 v_2|^2 \). We can easily find conditions to derive the orthogonal matrix \( U \) in eq.(12), which do not depend on the specific values of \( \tan \beta \), as follows:

\[ g_1 + g_2 + 2g - 2\xi = 0 , \quad g_1 = g_2 , \quad \phi = \frac{\pi}{4} . \quad (16) \]

The \( CP \) violating phase \( \phi \) takes its maximal value as is expected. The condition \( g_1 = g_2 \) may be reasonable since the infrared fixed point, approached using the renormalization group equations, suggests \( g_1 \simeq g_2[13] \). The condition of \( g_1 + g_2 + 2g - 2\xi = 0 \) gives an important constraint for the neutral Higgs scalars and the charged Higgs one, since \( \xi \) determines the charged Higgs mass as follows:

\[ m_{H_{ch}}^2 = \xi v^2 . \quad (17) \]

In addition to these constraints, we have the positivity condition expressed as[13]:

\[ g_1 > 0 , \quad g_2 > 0 , \quad h < 0 , \quad h + g' < 0 , \quad g + g' + h > -\sqrt{g_1 g_2} . \quad (18) \]

Under the condition of eq.(16), we have following matrix elements for \( U \):

\[ U_{11} = \cos \beta , \quad U_{21} = -\sin \beta , \]
\[ U_{12} = \cos \phi \sin \beta , \quad U_{22} = \cos \phi \cos \beta , \]
\[ U_{13} = \sin \phi \sin \beta , \quad U_{23} = \sin \phi \cos \beta , \quad (19) \]
with $\phi = \pi/4$. Then, the matrix elements of $O$ are given by eq.(9) as:

$$O_{11} = \cos 2\beta , \quad O_{12} = \sin \phi \sin 2\beta , \quad O_{13} = \cos \phi \sin 2\beta .$$\hspace{1cm} (20)

We have two solutions yielding maximal $CP$ violation, which satisfy the condition in eq.(6), for $\tan \beta$:

$$\tan \beta = \frac{1}{\sqrt{2}}(\sqrt{3} - 1) = 0.51 \cdots , \quad \tan \beta = \frac{1}{\sqrt{2}}(\sqrt{3} + 1) = 1.93 \cdots .$$\hspace{1cm} (21)

On the other hand, the masses of the three neutral Higgs scalars are given as

$$m_{H1}^2 = 2g_1 \cos^4 \beta + 2g_2 \sin^4 \beta + 4(\xi - g) \sin^2 \beta \cos^2 \beta = 2g_1 ,$$

$$m_{H2}^2 = g' + \xi + h , \quad m_{H3}^2 = g' + \xi - h ,$$\hspace{1cm} (22)

in units of $v^2$, where the conditions in eq.(16) are used in the second equality of $m_{H1}^2$. We notice that the four Higgs masses are given by four parameters $g_1$, $g'$, $h$ and $\xi$. Since the parameter $h$ is predicted to be very small in some analyses[13], the values of $m_{H2}$ and $m_{H3}$ are expected to be almost degenerate. The values of $m_{H2}$ and $m_{H3}$ are smaller than $m_{Hch}$ because $g' + h$ is negative as seen in eq.(18). On the other hand, $m_{H1}$ is not constrained in these discussions. However, we use 700 GeV as the upper bounds of the Higgs masses given by the perturbative analyses of THDM[14].

Let us study the $\rho$ parameter in the case of maximal $CP$ violation with the Higgs masses in eq.(22). We set the SM with one Higgs doublet($m_{Href}$) as a reference point and study the deviation from this point. The extra contribution to $\rho$ in the $CP$ violating THDM becomes[9,10]:

$$\Delta \rho = \frac{3\alpha}{16\pi \cos^2 \theta_w} \sum_{i=1}^{3} \frac{O_{ii}^2}{m_{Z_i}^2 - m_W^2} L(m_{Hi}^2, m_{Href}^2)$$

$$+ \frac{\alpha}{16\pi \sin^2 \theta_w m_W^2} \left[ \sum_{i=1}^{3} (1 - O_{ii})^2 F(m_{Hi}^2, m_{Hch}^2) - \frac{1}{2} \sum_{i,j,k=1 \atop i \neq j, j \neq k, k \neq i}^{3} O_{ii}^2 F(m_{Hj}^2, m_{Hk}^2) \right] ,$$\hspace{1cm} (23)
where
\[
F(x, y) = \frac{x + y}{2} - \frac{xy}{x - y} \log \frac{x}{y},
\]
\[
L(x, y) = F(x, m_Z^2) - F(x, m_W^2) + F(y, m_W^2) - F(y, m_Z^2).
\]

(24)

We present numerical results in the case of maximal $CP$ violation. The lower bound of the charged Higgs scalar mass has been obtained by studying the inclusive decay $B \rightarrow X_s \gamma[15]$, as to which the upper bound of the branching ratio was recently given by the CLEO collaboration[16]. The obtained lower bound is around 300GeV, which corresponds to $\xi > 3$. Therefore, we take the charged Higgs scalar mass to be larger than 300GeV. Then, the neutral Higgs scalar masses $m_{H1}$ and $m_{H3}$ are taken to be smaller than $m_{Hch}$, depending on the absolute values of $g'$ and $h$ as seen in eq.(22).

We show the $m_{Hch}$ dependence on $\Delta \rho$ taking four extreme parameter sets, (1)$g' = -0.5, h = -0.3, m_H1 = 100GeV$, (2)$g' = -2, h = -0.3, m_H1 = 100GeV$, (3)$g' = -0.5, h = -0.3, m_H1 = 700GeV$ and (4)$g' = -2, h = -0.3, m_H1 = 700GeV$, in fig.1, where the recent experimental value $\rho = (3.0 \pm 1.7) \times 10^{-3}[17]$ is denoted by the horizontal dotted lines. Here, we take $m_{Href} = m_Z$ as a reference point. The magnitude of $h$ is taken to be rather larger than usual[13] in order to protect the under-estimate of $\Delta \rho$. We consider two cases, that the value of $|g'|$ is small and that it is large in order to find the $g'$ dependence of our predictions. As seen in fig.1, $\Delta \rho$ does not vary greatly relative to the experimental value, and that our predictions are almost all negative except for the case (2)$g' = -2, h = -0.3, m_H1 = 100GeV$, in which we have a rather light neutral Higgs with a mass of 100GeV.
In order to find the $m_{H1}$ dependence of our result, we present $\Delta\rho$ versus $m_{H1}$ in four extreme cases, $(5) g' = -0.5$, $h = -0.3$, $m_{Hch} = 350\text{GeV}$, $(6) g' = -2$, $h = -0.3$, $m_{Hch} = 350\text{GeV}$, $(7) g' = -0.5$, $h = -0.3$, $m_{Hch} = 700\text{GeV}$ and $(8) g' = -2$, $h = -0.3$, $m_{Hch} = 700\text{GeV}$, in fig.2. We found that our prediction is not large relative to the experimental value unless $m_{H1}$ is lower than 100GeV. Thus, the THDM with maximal $CP$ violation is not contradicted with the present data of the $\rho$ parameter due to the conditions in eq.(16) unless the extremely large value of $|g'|$ is taken, even if the custodial symmetry is absent in the Higgs Lagrangian.

It may be useful to comment on the $CP$ violating parameter $\text{Im}Z_i^{(k)}$, which is the imaginary part of the $k$-th column vectors in the neutral Higgs scalar vector space, defined in ref.11. For the first Higgs scalar, these are zero because the third component of the eigenvector is zero as seen in eq.(12), i.e., there is no scalar-pseudoscalar interference term. We have non-vanishing values for second and third Higgs scalars ($k=2,3$) as follows:

$$\text{Im}Z_1^{(2)} = -\text{Im}Z_1^{(3)} = \frac{1}{4}(\sqrt{3} \mp 1)^2, \quad \text{Im}Z_2^{(2)} = -\text{Im}Z_2^{(3)} = -\frac{1}{4}(\sqrt{3} \pm 1)^2, \quad (25)$$

the signs $\pm$ correspond to the two solutions of $\tan\beta$ in eq.(21). We notice that these values are somewhat smaller than the Weinberg’s bound[11] taking the same value of $\tan\beta$ where

$$\left|\text{Im}Z_1^{(2,3)}\right| \sim \left\{ \begin{array}{ll} 0.89 \quad \text{Im}Z_1^{(WB)} \\ 0.46 \quad \text{Im}Z_2^{(WB)} \end{array} \right., \quad (26)$$

(WB) denoting the Weinberg’s bounds, and the upper values and lower ones correspond
to the two solutions of $\tan \beta$. Thus, the Weinberg’s bound does not correspond to maximal $CP$ violation.

We summarize as follows. We have studied the conditions necessary to give maximal $CP$ violation in THDM with the $CP$ violation. We obtained fixed values of $\tan \beta$, and constraints on the Higgs potential parameters. Two neutral Higgs scalars must be lighter than the charged Higgs scalar and these two Higgs scalars are expected to be almost degenerate due to the smallness of the parameter $h$. Under these conditions, the contribution of the Higgs scalar exchanges to the $\rho$ parameter is rather small and negative in a wide range of Higgs masses. Thus, maximum $CP$ violation in THDM is safely allowed for the $\rho$ parameter without the custodial symmetry.

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Figure Captions

**Fig.1:** The contributions of the Higgs scalar exchanges to the $\rho$ parameter versus the charged Higgs scalar mass. The cases (1), (2), (3) and (4) are denoted by the solid, dotted, dashed, dashed-dotted curves, respectively. The horizontal dashed lines denote the experimental bounds.

**Fig.2:** The contributions of the Higgs scalar exchanges to the $\rho$ parameter versus $m_{H_1}$. The cases (5), (6), (7) and (8) are denoted by the solid, dotted, dashed, dashed-dotted curves, respectively.
This figure "fig1-1.png" is available in "png" format from: