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Fermion Masses and Mixings in a String Inspired Model

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Abstract

In the context of Calabi-Yau string models we explore the origin of characteristic pattern of quark-lepton masses and the CKM matrix. The discrete R-symmetry $Z_K \times Z_2$ is introduced and the Z_2 is assigned to the R-parity. The gauge symmetry at the string scale, $SU(6) \times SU(2)_R$, is broken into the standard model gauge group at a very large intermediate energy scale. At energies below the intermediate scale down-type quarks and also leptons are mixed with unobserved heavy states, respectively. On the other hand, there are no such mixings for up-type quarks. Due to the large mixings between light states and heavy ones we can derive phenomenologically viable fermion mass hierarchies and the CKM matrix. Mass spectra for intermediate-scale matter beyond the MSSM are also determined. Within this framework proton lifetime is long enough to be consistent with experimental data. As for the string scale unification of gauge couplings, however, consistent solutions are not yet found.

1 Introduction

In order to make sure of the reality of string theory it is important to explore how string theory determines low-energy parameters which are free parameters in the standard model and in the minimal supersymmetric standard model (MSSM). Among many issues of low-energy parameters, the characteristic pattern of quark-lepton masses and mixing angles has long been a challenging problem to explain its origin. The observed masses of quarks and leptons have the hierarchical pattern

(i) $$m(1\,\mathrm{st\ gen.}) \ll m(2\,\mathrm{nd\ gen.}) \ll m(3\,\mathrm{rd\ gen.})$$

and also the ratios among quark masses are in line as

(ii) $$m_u/m_d < m_c/m_s < m_t/m_b.$$

Up to now several possibilities of explaining these features have been studied by many authors [1]-[7]. A possibility is that all the observed pattern of fermion masses are attributable to the boundary condition, i.e. to the hierarchical structure of Yukawa couplings themselves at a very large scale. However, when we take GUT-type models, it is difficult to find a satisfactory solution in which property (ii) comes into line with a simple unification of Yukawa couplings. In this paper we explore a somewhat distinct possibility. In the context of Calabi-Yau string models with Kac-Moody level-one we propose a new type of model which potentially generates the characteristic pattern of fermion masses and Cabibbo-Kobayashi-Maskawa(CKM) matrix. In the model property (i) is attributed to the texture of renormalizable and nonrenormalizable interactions restricted by some discrete symmetries at the string scale. This mechanism is similar to those proposed in Refs. [1][2][3][4]. On the other hand, property (ii) comes from large mixings among states observed at low energies and unobserved heavy ones. The mixings occur only for down-type quarks and for leptons below the energy scale at which the gauge group is broken into the standard model gauge group $G_{st} = SU(3)_c \times SU(2)_L \times U(1)_Y$.

The four-dimensional effective theory from Calabi-Yau string compactification is far more constrained than ordinary field theory. In the effective theory there are many peculiar features beyond the MSSM. First point is that the gauge group G, which is given via the flux breaking at the string scale M_S, is a subgroup of E_6 and would be larger than the standard model gauge group G_{st}. We will choose $G = SU(6)\times SU(2)_R$, under which doublet Higgs and color-triplet Higgs fields transform differently [8]. As we will see later, the gauge group G is spontaneously broken to G_{st} in two steps at very large intermediate energy scales. Second point is that the massless sector of the Calabi-Yau string model contains extra particles beyond the MSSM. In string inspired models we typically have a number of generations and anti-generations. For illustration, if the gauge group G is E_6, the massless chiral superfields apart from E_6-singlets consist of

$$N_f\,\mathbf{27} \ + \ \delta\,(\mathbf{27} + \mathbf{27}^*)\,, \tag{1.1}$$

where N_f means the family number at low energies. It should be noted that δ sets of vector-like multiplets are included in the massless sector. In Calabi-Yau string compactification the generation structure of matter fields is closely linked to the topological structure of the compactified manifold. We will assume $\delta = 1$ for the sake of simplicity. Adjoint Higgs representations which are introduced in the traditional GUT are not allowed at Kac-Moody level-one. In addition, particles beyond the MSSM are contained also in $\mathbf{27}$-representation of E_6. Namely in $\mathbf{27}$ we have quark superfields $Q = (U, D)$, U^c, D^c, lepton superfields $L = (N, E)$, N^c, E^c, Higgs doublets H_u, H_d, color-triplet Higgses g, g^c and an $SO(10)$-singlet S. When the gauge group G is broken into G_{st}, superfields D^c and g^c as well as L and H_d are indistinguishable from each other under G_{st}. Hence there possibly appear mixings between D^c and g^c and between L and H_d. On the other hand, for up-type quarks there appear no such mixings. While up-type, down-type quarks and leptons share their interactions in common at the string scale M_S, D^c-g^c mixing and L-H_d mixing potentially turn mass pattern of down-type quarks and leptons out of that of up-type quarks at low energies. Further the mixings may be responsible for the CKM matrix. Third point of peculiar features beyond the MSSM is that superstring theory naturally provides the discrete symmetries which stem from symmetric structure of the compactified space. As shown in Gepner model [9], the discrete symmetry can be the R-symmetry under which the components of a given superfield transform differently. Also, the discrete symmetry could be used as a horizontal symmetry. The discrete R-symmetry strongly limits the renormalizable and nonrenormalizable interactions and then possibly controls parameters in the low-energy effective theory. Recently it has been argued that the discrete R-symmetry controls energy scales of the symmetry breaking [10], the magnitude of Majorana masses of the right-handed neutrino [11] and the stability of the weak-scale hierarchy [12]. We will introduce the discrete R-symmetry $Z_K \times Z_2$ at the string scale.

In this paper main emphasis is placed on how both of the mixing mechanism mentioned above and the discrete symmetry bring about phenomenologically viable fermion mass pattern and the CKM matrix. This paper is organized as follows. In section 2 we introduce the discrete R-symmetry, which puts stringent constraints on interactions in the superpotential. The Z_2 symmetry is chosen so as to be in accord with the so-called R-parity in the MSSM. R-parity is conserved over the whole energy range from the string scale to the electroweak scale. After arguing that the discrete R-symmetry controls energy scales of the gauge symmetry breaking, we study particle spectra of vector-like multiplets in section 3. Since doublet Higgses and color-triplet Higgses belong to the different representations of the gauge group

$G = SU(6) \times SU(2)_R$, distinct particle spectra of these fields are derived without some fine-tuning of parameters. In section 4 mass matrices for colored chiral multiplets are presented. There appear mixings between D^c and g^c. Choosing appropriate assignments of discrete charges, we get large mixings between them. Due to the maximal mixing the mass pattern of down-type quarks differs from that of up-type quarks. The model generates not only the hierarchical pattern of quark masses but also the texture of the CKM matrix. In section 5 we discuss mixings between L and H_d and study spectra of leptons. The CKM matrix for leptons turns out to be a unit matrix. In the present framework we have several R-parity even colored superfields which potentially mediate proton decay. In section 6 it is shown that proton lifetime is about $10^{33 \sim 35}$year. In section 7 we find that the gauge coupling unification is not successfully achieved as a consequence of spectra of extra intermediate-scale matter. In the final section we conclude with a brief summary of our results. In Appendix A it is shown that under an appropriate condition on the soft SUSY breaking parameters the gauge symmetry is broken at tree level. In Appendix B we show that if neutrino Majorana masses are sufficiently large compared with the soft supersymmetry(SUSY) breaking scale $m_{3/2} = O(1\mathrm{TeV})$, the scalar potential is minimized along the direction where R-parity is conserved.

2 Discrete R-symmetry

In order to guarantee the stability of the weak-scale hierarchy without fine-tuning, it is favorable that doublet Higgses and color-triplet Higgses reside in different irreducible representations of the string scale gauge group G. As the largest gauge group implementing such a situation is $G = SU(6) \times SU(2)_R$ [8], in this paper we choose $SU(6) \times SU(2)_R$ as an example of G. Chiral superfields (Φ) in **27** representation of

E_6 are decomposed into

$$\begin{aligned}\Phi(\mathbf{15},\mathbf{1}) &: \quad Q, L, g, g^c, S, \\ \Phi(\mathbf{6}^*,\mathbf{2}) &: \quad U^c, D^c, N^c, E^c, H_u, H_d.\end{aligned}$$

Although L and H_d (D^c and g^c) have the same quantum numbers under G_{st}, they belong to different irreducible representations of $SU(6)\times SU(2)_R$. The superpotential W is described in terms of $\mathbf{27}$ chiral superfields (Φ) and $\mathbf{27}^*$ ones ($\overline{\Phi}$) as

$$W = \Phi^3 + \overline{\Phi}^3 + (\Phi\overline{\Phi})^{m+1} + \Phi^3(\Phi\overline{\Phi})^n + \cdots, \tag{2.1}$$

where m and n are positive integers and all the terms are characterized by the couplings of $O(1)$ in units of $M_S = O(10^{18}\mathrm{GeV})$. The cubic term Φ^3 is of the forms

$$(\Phi(\mathbf{15},\mathbf{1}))^3 = QQg + Qg^cL + g^cgS, \tag{2.2}$$

$$\begin{aligned}\Phi(\mathbf{15},\mathbf{1})(\Phi(\mathbf{6}^*,\mathbf{2}))^2 &= QH_dD^c + QH_uU^c + LH_dE^c + LH_uN^c \\ &\quad + SH_uH_d + gN^cD^c + gE^cU^c + g^cU^cD^c.\end{aligned} \tag{2.3}$$

We assume that the massless matter fields are composed of chiral multiplets Φ_i ($i = 1, \cdots, N_f = 3$) and a set ($\delta = 1$) of vector-like multiplets Φ_0 and $\overline{\Phi}$. Here we introduce the discrete R-symmetry $Z_K \times Z_2$ as a stringy selection rule. As we will see below, large K is favorable for explaining the mass pattern of quarks and leptons. The Z_2 symmetry is taken so as to be in accord with the R-parity in the MSSM. Therefore, hereafter the Z_2 symmetry is referred to as R-parity. Supposing that ordinary quarks and leptons are included in chiral multiplets Φ_i ($i = 1, 2, 3$), R-parity of all Φ_i ($i = 1, 2, 3$) are set to be odd. Since light Higgs scalars are even under R-parity, light Higgs doublets are bound to reside in Φ_0 and/or $\overline{\Phi}$. For this reason we assign even R-parity to Φ_0 and $\overline{\Phi}$. In Appendix B we show that once the R-parity is conserved at the string scale, the R-parity remains unbroken down to the electroweak scale under appropriate conditions. Hence, through the spontaneous breaking of gauge symmetry gauge superfields are possibly mixed with the vector-like multiplets Φ_0 and $\overline{\Phi}$ but not with the chiral multiplets Φ_i $(i = 1, 2, 3)$. Furthermore, no mixing occurs between the vector-like multiplets and the chiral multiplets.

We use the Z_K symmetry as a horizontal symmetry and construct our model incorporating the mechanism of Froggatt and Nielsen[1]. The Z_K symmetry controls not only a large hierarchy of the energy scales of the symmetry breaking but also the texture of effective Yukawa couplings. We denote the Z_K-charges of chiral multiplets $\Phi_i(\mathbf{15}, \mathbf{1})$ and $\Phi_i(\mathbf{6}^*, \mathbf{2})$ by a_i and b_i $(i = 0, 1, 2, 3)$, respectively. In Table I, we tabulate the notations for Z_K-charges and the assignment of R-parity for each superfield. Note that the anticommuting number θ has also a $Z_K \times Z_2$-charge $(-1, -)$.

Table I

3 Gauge hierarchy and the μ-term

The discrete symmetry introduced above puts stringent constraints on both renormalizable and nonrenormalizable interactions in the superpotential. To begin with, Z_K-charges of vector-like multiplets are chosen such that both the nonrenormalizable interactions

$$\left(\Phi_0(\mathbf{15}, \mathbf{1})\overline{\Phi}(\mathbf{15}^*, \mathbf{1})\right)^{sk} \tag{3.1}$$

and

$$\left(\Phi_0(\mathbf{6}^*, \mathbf{2})\overline{\Phi}(\mathbf{6}, \mathbf{2})\right)^{s} \tag{3.2}$$

are allowed, where $K = sk + 1$ and s and k are even and odd integers larger than unity, respectively. This implies that $sk(a_0 + \overline{a}) + 2 \equiv s(b_0 + \overline{b}) + 2 \equiv 0$ in modulus

$K = sk + 1$. Thus we impose

$$a_0 + \overline{a} \equiv 2, \qquad b_0 + \overline{b} \equiv 2k \qquad \mathrm{mod}\ K. \tag{3.3}$$

It follows that the interactions

$$W_{SN} \sim \sum_{r=0}^{s} (\Phi_0(\mathbf{15}, \mathbf{1})\overline{\Phi}(\mathbf{15}^*, \mathbf{1}))^{(s-r)k} (\Phi_0(\mathbf{6}^*, \mathbf{2})\overline{\Phi}(\mathbf{6}, \mathbf{2}))^r \tag{3.4}$$

are allowed in M_S units. Due to R-parity conservation the interactions containing even number of Φ_i ($i = 1, 2, 3$) are also allowed but all of the interactions containing odd number of Φ_i ($i = 1, 2, 3$) are forbidden.

Incorporating the soft SUSY breaking terms together with the F- and D-terms, we get the scalar potential. Although dynamics of SUSY breaking is not presently known, we may parametrize the SUSY breaking by introducing the universal soft terms. The scale of SUSY breaking $m_{3/2}$ is supposed to be $O(1\mathrm{TeV})$. Through the minimization of the scalar potential we are able to detemine a ground state, which is characterized by VEVs of Φ_0, $\overline{\Phi}$ and Φ_i ($i = 1, 2, 3$). Under appropriate conditions on soft SUSY breaking parameters the gauge symmetry is spontaneously broken at tree level (see Appendix A). Further, if masses of G_{st}-neutral and R-parity odd superfields are sufficiently larger than $m_{3/2}$, the scalar potential is minimized at vanishing $\langle \Phi_i \rangle$ for $i = 1, 2, 3$ (see Appendix B). On the other hand, Φ_0 and $\overline{\Phi}$ acquire nonzero VEVs along a D-flat direction, namely

$$\langle \Phi_0(\mathbf{15}, \mathbf{1}) \rangle = \langle \overline{\Phi}(\mathbf{15}^*, \mathbf{1}) \rangle \simeq M_S\, x, \tag{3.5}$$

$$\langle \Phi_0(\mathbf{6}^*, \mathbf{2}) \rangle = \langle \overline{\Phi}(\mathbf{6}, \mathbf{2}) \rangle \simeq M_S\, x^k \tag{3.6}$$

up to phase factors [10][11], where

$$x = \left(\frac{m_{3/2}}{M_S} \right)^{1/(2sk-2)}. \tag{3.7}$$

Although for a large K the parameter x by itself is not a very small number, the large hierarchy occurs by raising the number to large powers. Hence, x becomes an efficient parameter in describing the hierarchical structure of the effective theory. Note that we have the inequalities

$$M_S > |\langle \Phi_0(\mathbf{15}, \mathbf{1}) \rangle| > |\langle \Phi_0(\mathbf{6}^*, \mathbf{2}) \rangle| \gg \sqrt{m_{3/2} M_S}. \tag{3.8}$$

Hereafter the fields $\Phi_0(\mathbf{15}, \mathbf{1})$ and $\overline{\Phi}(\mathbf{15}^*, \mathbf{1})$ which develop non-zero VEVs are referred to as G_{st}-neutral fields S_0 and $\overline{S}$, respectively. At the scale $\langle S_0 \rangle = \langle \overline{S} \rangle \simeq M_S\, x$ the gauge symmetry $SU(6) \times SU(2)_R$ is spontaneously broken to $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, where $SU(4)_{PS}$ stands for the Pati-Salam $SU(4)$ [13]. Under the $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ the chiral superfields $\Phi(\mathbf{15}, \mathbf{1})$ and $\Phi(\mathbf{6}^*, \mathbf{2})$ are decomposed as

$$(\mathbf{15}, \mathbf{1}) \quad = \quad (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}), \tag{3.9}$$

$$(\mathbf{6}^*, \mathbf{2}) \quad = \quad (\mathbf{4}^*, \mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}), \tag{3.10}$$

where each matter field is assigned as

$$\begin{array}{rcl}
\Phi(\mathbf{4}, \mathbf{2}, \mathbf{1}) & : & Q, L, \\
\Phi(\mathbf{6}, \mathbf{1}, \mathbf{1}) & : & g, g^c, \\
\Phi(\mathbf{1}, \mathbf{1}, \mathbf{1}) & : & S, \\
\Phi(\mathbf{4}^*, \mathbf{1}, \mathbf{2}) & : & U^c, D^c, N^c, E^c, \\
\Phi(\mathbf{1}, \mathbf{2}, \mathbf{2}) & : & H_u, H_d.
\end{array}$$

The subsequent symmetry breaking takes place via the non-zero VEVs $\langle \Phi_0(\mathbf{6}^*, \mathbf{2}) \rangle = \langle \overline{\Phi}(\mathbf{6}, \mathbf{2}) \rangle \simeq M_S\, x^k$. At this stage of the symmetry breaking there seem to be two possibilities depending on whether the non-zero VEV $\langle \Phi_0(\mathbf{6}^*, \mathbf{2}) \rangle (\langle \overline{\Phi}(\mathbf{6}, \mathbf{2}) \rangle)$ is attributed to $\langle \Phi_0(\mathbf{4}^*, \mathbf{1}, \mathbf{2}) \rangle (\langle \overline{\Phi}(\mathbf{4}, \mathbf{1}, \mathbf{2}) \rangle)$ or $\langle \Phi_0(\mathbf{1}, \mathbf{2}, \mathbf{2}) \rangle (\langle \overline{\Phi}(\mathbf{1}, \mathbf{2}, \mathbf{2}) \rangle)$. As will be seen soon later, we have the term $(S_0 \overline{S})^p \overline{S} \overline{H}_u \overline{H}_d$ in the superpotential, where p is a positive integer detemined by the discrete symmetry Z_K. Under an appropriate charge assignment of matter fields we have $p \simeq sk - 2k$. According to the presence of this superpotential term the large VEV $\langle\overline{\Phi}(\mathbf{1},\mathbf{2},\mathbf{2})\rangle$ is inconsistent with the (almost) F-flat condition. Consequently, the subsequent symmetry breaking occurs through $\langle\Phi_0(\mathbf{4}^*,\mathbf{1},\mathbf{2})\rangle = \langle\overline{\Phi}(\mathbf{4},\mathbf{1},\mathbf{2})\rangle \simeq M_S\, x^k$. Then we denote the fields $\Phi_0(\mathbf{4}^*,\mathbf{1},\mathbf{2})$ and $\overline{\Phi}(\mathbf{4},\mathbf{1},\mathbf{2})$ with the non-zero VEVs as N_0^c and $\overline{N}^c$, respectively. Thus the gauge symmetry is spontaneously broken in two steps at the scales $\langle S_0\rangle$ and $\langle N_0^c\rangle$ as

$$
\begin{aligned}
SU(6)\times SU(2)_R \;&\xrightarrow{\langle S_0\rangle}\; SU(4)_{PS}\times SU(2)_L\times SU(2)_R &(3.11)\\
&\xrightarrow{\langle N_0^c\rangle}\; SU(3)_c\times SU(2)_L\times U(1)_Y. &(3.12)
\end{aligned}
$$

Since S_0, $\overline{S}$, N_0^c and $\overline{N}^c$ acquire VEVs along a D-flat direction, SUSY is maintained down to $O(1\mathrm{TeV})$.

At the first step of the symmetry breaking chiral superfields Q_0, L_0, $\overline{Q}$, $\overline{L}$ and $(S_0-\overline{S})/\sqrt{2}$ are absorbed by gauge superfields. Through the subsequent symmetry breaking chiral superfields U_0^c, E_0^c, $\overline{U}^c$, $\overline{E}^c$ and $(N_0^c-\overline{N}^c)/\sqrt{2}$ are absorbed. On the other hand, for components $(S_0+\overline{S})/\sqrt{2}$ and $(N_0^c+\overline{N}^c)/\sqrt{2}$ the mass matrix is of the form

$$
\begin{pmatrix} O(x^{2sk-2}) & O(x^{(2s-1)k-1}) \\ O(x^{(2s-1)k-1}) & O(x^{2(s-1)k}) \end{pmatrix} \tag{3.13}
$$

in M_S units. This yields mass eigenvalues

$$
O(m_{3/2}), \quad O(M_S x^{2(s-1)k}), \tag{3.14}
$$

which correspond to the eigenstates

$$
\begin{aligned}
&\tfrac{1}{\sqrt{2}}(S_0+\overline{S}) + O(x^{k-1})\tfrac{1}{\sqrt{2}}(N_0^c+\overline{N}^c),\\
&\tfrac{1}{\sqrt{2}}(N_0^c+\overline{N}^c) + O(x^{k-1})\tfrac{1}{\sqrt{2}}(S_0+\overline{S}),
\end{aligned} \tag{3.15}
$$

respectively [11]. The discrete symmetry Z_K is broken together with $SU(6)\times SU(2)_R$ by the VEV $\langle S_0\rangle$, while the VEV allows the Z_2-symmetry (referred to R-parity conservation) to remain unbroken all the way down to TeV.

In order to stabilize the weak-scale hierarchy we put an additional requirement that the interaction

$$(S_0\overline{S})^{sk-e} S_0 H_{u0} H_{d0} \tag{3.16}$$

is allowed with $e = 0, 1$ in the superpotential. We will shortly show that the μ-problem is solved by this setting $e = 0, 1$. This condition is translated into

$$a_0 + 2b_0 \equiv 2e \mod K. \tag{3.17}$$

From Eqs.(3.3) and (3.17) the superpotential of Higgs doublet in vector-like multiplets has the form

$$\begin{aligned} W_H \quad \sim \quad & (S_0\overline{S})^{(s-2)k+e-1}\overline{S}\,\overline{H}_u\overline{H}_d + (S_0\overline{S})^{(s-1)k}(H_{u0}\overline{H}_u + H_{d0}\overline{H}_d) \\ & + (S_0\overline{S})^{sk-e} S_0 H_{u0} H_{d0}. \end{aligned} \tag{3.18}$$

When S_0 and $\overline{S}$ develop the non-zero VEVs, the superpotential induces the mass matrix of H_{u0}, H_{d0}, $\overline{H}_u$ and $\overline{H}_d$

$$\begin{array}{cc} & \begin{array}{cc} \quad\overline{H}_u \quad\quad\quad & \quad\quad H_{d0} \end{array} \\ \begin{array}{c} \overline{H}_d \\ H_{u0} \end{array} & \left(\begin{array}{cc} O(x^{2(s-2)k+2e-1}) & O(x^{2(s-1)k}) \\ O(x^{2(s-1)k}) & O(x^{2sk-2e+1}) \end{array} \right) \end{array} \tag{3.19}$$

in M_S units, which leads to the mass eigenvalues

$$O(M_S\, x^{2(s-2)k+2e-1}), \quad O(M_S\, x^{2sk-2e+1}) = O(m_{3/2}\, x^{3-2e}). \tag{3.20}$$

Consequently, we have the μ-term with $m_{3/2} > \mu = O(m_{3/2}\, x^{3-2e}) = O(10^{2\sim 3}\mathrm{GeV})$ for $e = 0, 1$ [8][12]. Here, note that we take $x \sim 0.7$ with $sk = 50$ in a typical example given later. Light Higgs states are given by

$$H_{u0} + O(x^{2k+1-2e})\overline{H}_d, \qquad H_{d0} + O(x^{2k+1-2e})\overline{H}_u. \tag{3.21}$$

The components of $\overline{H_d}$ and $\overline{H_u}$ in light Higgses are small. Generally speaking, in the superpotential W_H there exist additional terms which are obtained by replacing each factor $(S_0\overline{S})^k$ by a factor $(N_0^c\overline{N}^c)$. However, as far as the mass matrices are concerned, these terms yield the same order of magnitude as in each entry of the above matrix because of the relations $(|\langle S_0\rangle|/M_S)^k = |\langle N_0^c\rangle|/M_S$ and $k(a_0+\overline{a}) \equiv b_0+\overline{b}$. Since we do not address here the issue of CP-violation, all VEVs are assumed to be real for simplicity. Therefore, hereafter the nonrenormalizable terms are expressed in terms only of the powers of $(S_0\overline{S})$. Note that the product $H_{u0}H_{d0}$ has a nonzero Z_K-charge. In contrast with the present model, in a solution of the μ-problem proposed in Ref.[14] the R-charge of the product of light Higgses has to be zero.

The remaining components in Φ_0 and $\overline{\Phi}$, i.e. g_0, g_0^c, D_0^c and $\overline{g}$, $\overline{g}^c$, $\overline{D}^c$ are down-type color-triplet fields. In the present model the spectra of color-triplet Higgses are quite different from those of doublet Higgses. Mass matrix for these fields is given in section 6.

4 Quark masses and the CKM matrix

Next we turn to mass matrices for chiral multiplets Φ_i $(i=1,2,3)$. Due to R-parity conservation Φ_i $(i=1,2,3)$ are not mixed with vector-like multiplets Φ_0 and $\overline{\Phi}$. The superpotential of up-type quarks which contributes to the mass matrix of up-type quarks, is given by

$$W_U \sim (S_0\overline{S})^{m_{ij}} Q_i U_j^c H_{u0} \qquad (i,j=1,2,3), \tag{4.1}$$

where the exponents m_{ij} are integers in the range $0 \leq m_{ij} < K = sk+1$. Although the Z_K symmetry allows the terms multiplied by $(S_0\overline{S})^K$, $(S_0\overline{S})^{2K}$, $\cdots$, the contributions of these terms are negligibly small compared with the above ones. Therefore, it is sufficient for us to take only the terms with $m_{ij} < K$. Recall that light Higgs doublets are almost H_{u0} and H_{d0}. Under the Z_K-symmetry the exponent m_{ij} is determined by the condition

$$2m_{ij} + a_i + b_j + b_0 + 2 \equiv 0 \qquad \bmod K. \tag{4.2}$$

Instead of a_i and b_i $(i = 1, 2, 3)$, hereafter we introduce new notations α, β, γ, and δ defined by

$$a_2 - a_1 \equiv \alpha, \quad b_2 - b_1 \equiv \beta, \quad a_3 - a_2 \equiv \gamma, \quad b_3 - b_2 \equiv \delta. \tag{4.3}$$

These parameters are supposed to be even integers to derive desirable mass pattern of quarks and

$$0 < \alpha \leq \delta \leq \gamma \leq \beta, \qquad 2(\beta + \delta) < K. \tag{4.4}$$

The above condition (4.2) is rewritten as

$$2m_{ij} \equiv 2m_{33} + \begin{pmatrix} \alpha+\beta+\gamma+\delta & \alpha+\gamma+\delta & \alpha+\gamma \\ \beta+\gamma+\delta & \gamma+\delta & \gamma \\ \beta+\delta & \delta & 0 \end{pmatrix}_{ij} \qquad \mathrm{mod}\ K, \tag{4.5}$$

where $2m_{33} \equiv -a_3 - b_3 - b_0 - 2$. The mass matrix of up-type quarks is described by a 3×3 matrix M with elements

$$M_{ij} = O(x^{2m_{ij}}) \tag{4.6}$$

multiplied by $v_u = \langle H_{u0} \rangle$. This equation is an order of magnitude relationship, so that each element will be multiplied by an $O(1)$ number. From Eq.(4.5) the matrix M is generally asymmetric. Here we take an ansatz that only top-quark has a trilinear coupling. This means that

$$m_{33} = 0. \tag{4.7}$$

When we adopt appropriate unitary matrices $\mathcal{V}_u$ and $\mathcal{U}_u$, the matrix

$$\mathcal{V}_u^{-1} M \, \mathcal{U}_u \tag{4.8}$$

becomes diagonal. Explicitly, $\mathcal{V}_u$ and $\mathcal{U}_u$ are of the forms

$$\mathcal{V}_u = \begin{pmatrix} 1 - O(x^{2\alpha}) & O(x^{\alpha}) & O(x^{\alpha+\gamma}) \\ O(x^{\alpha}) & 1 - O(x^{2\alpha}) & O(x^{\gamma}) \\ O(x^{\alpha+\gamma}) & O(x^{\gamma}) & 1 - O(x^{2\gamma}) \end{pmatrix}, \tag{4.9}$$

$$\mathcal{U}_u = \begin{pmatrix} 1 - O(x^{2\beta}) & O(x^{\beta}) & O(x^{\beta+\delta}) \\ O(x^{\beta}) & 1 - O(x^{2\delta}) & O(x^{\delta}) \\ O(x^{\beta+\delta}) & O(x^{\delta}) & 1 - O(x^{2\delta}) \end{pmatrix}. \tag{4.10}$$

The eigenvalues of M are

$$O(x^{\alpha+\beta+\gamma+\delta}), \qquad O(x^{\gamma+\delta}), \qquad O(1), \tag{4.11}$$

which correspond to u-, c- and t-quarks, respectively.

Under $SU(6) \times SU(2)_R$ gauge symmetry down-type quarks and leptons share the nonrenormalizable terms in common with up-type quarks. Namely we get

$$W \sim (S_0\overline{S})^{m_{ij}} \{Q_i D_j^c H_{d0} + L_i N_j^c H_{u0} + L_i E_j^c H_{d0}\}. \tag{4.12}$$

For down-type quarks, however, the mixings between g^c and D^c should be taken into account at energies below the scale $\langle N_0^c \rangle$. This is because we have two down-type $SU(2)_L$-singlet colored fields in each **27** of E_6. Then, hereafter we denote R-parity odd g_i and g_i^c $(i = 1, 2, 3)$ as D'_i and D'^c_i $(i = 1, 2, 3)$, respectively. The superpotential of down-type colored fields is of the form

$$W_D \sim (S_0\overline{S})^{z_{ij}} S_0 D'_i D'^c_j + (S_0\overline{S})^{m_{ij}} (N_0^c D'_i + H_{d0} Q_i) D^c_j, \tag{4.13}$$

where the exponents z_{ij} are determined by

$$2z_{ij} + a_i + a_j + a_0 + 2 \equiv 0 \qquad \mathrm{mod}\ K \tag{4.14}$$

in the range $0 \leq z_{ij} < K$. Thus we have

$$2z_{ij} \equiv 2z_{33} + \left(\begin{array}{ccc} 2\alpha + 2\gamma & \alpha + 2\gamma & \alpha + \gamma \\ \alpha + 2\gamma & 2\gamma & \gamma \\ \alpha + \gamma & \gamma & 0 \end{array} \right)_{ij} \qquad \mathrm{mod}\ K \tag{4.15}$$

with $2z_{33} \equiv -2a_3 - a_0 - 2$. In terms of a 3×3 matrix Z with elements

$$Z_{ij} = O(x^{2z_{ij}}), \tag{4.16}$$

a mass matrix of down-type colored fields is written as

$$\widehat{M}_d = \begin{array}{c} \\ D' \\ D \end{array} \!\!\! \begin{array}{c} \begin{array}{cc} D'^c & D^c \end{array} \\ \left(\begin{array}{cc} xZ & x^k M \\ 0 & \rho_d M \end{array} \right) \end{array} \tag{4.17}$$

in M_S units below the scale $\langle N_0^c \rangle$, where $\rho_d = \langle H_{d0} \rangle / M_S = v_d / M_S$. This $\widehat{M}_d$ is a 6×6 matrix and can be diagonalized by a bi-unitary transformation as

$$\widehat{\mathcal{V}}_d^{-1} \widehat{M}_d \widehat{\mathcal{U}}_d. \tag{4.18}$$

$\widehat{M}_d$ shows mixings between D'^c and D^c, explicitly. This type of mixings does not occur for up-type quarks. From Eqs. (4.17) and (4.18) the matrix

$$\widehat{\mathcal{V}}_d^{-1} \widehat{M}_d \widehat{M}_d^\dagger \widehat{\mathcal{V}}_d = \widehat{\mathcal{V}}_d^{-1} \left(\begin{array}{cc} A_d + B_d & \epsilon_d B_d \\ \epsilon_d B_d & \epsilon_d^2 B_d \end{array} \right) \widehat{\mathcal{V}}_d \tag{4.19}$$

is diagonal, where

$$A_d = x^2 Z Z^\dagger, \qquad B_d = x^{2k} M M^\dagger, \qquad \epsilon_d = \rho_d \, x^{-k}. \tag{4.20}$$

In view of the smallness of the parameter ϵ_d, we use the perturbative method in solving the eigenvalue problem. It follows that the eigen equation is approximately separated into two pieces. For heavy states the eigen equation becomes

$$\det \left(A_d + B_d - \frac{\eta}{M_S^2} \right) = 0. \tag{4.21}$$

Solving this equation of a variable η, we obtain masses squared for three heavy states. The other three states are light and their masses are given by solving the eigen equation

$$\det \left(x^{-2k} (A_d^{-1} + B_d^{-1})^{-1} - \frac{\eta}{v_d^2} \right) = 0. \tag{4.22}$$

This equation is derived in ϵ_d^2 order of the perturbative expansion. The light states correspond to observed down-type quarks. If the mixing between D'^c and D^c is sizable, mass pattern of down-type quarks is possibly changed from that of up-type quarks. Thus in our model, property (ii) pointed out in section 1 for observed fermion masses is attributable to this mixing mechanism.

The 6×6 unitary matrices $\widehat{\mathcal{V}}_d$ and $\widehat{\mathcal{U}}_d$ are

$$\widehat{\mathcal{V}}_d \simeq \begin{pmatrix} \mathcal{W}_d & -\epsilon_d (A_d + B_d)^{-1} B_d \mathcal{V}_d \\ \epsilon_d B_d (A_d + B_d)^{-1} \mathcal{W}_d & \mathcal{V}_d \end{pmatrix}, \tag{4.23}$$

$$\widehat{\mathcal{U}}_d \simeq \begin{pmatrix} x Z^\dagger \mathcal{W}_d \, (\Lambda_d^{(0)})^{-1/2} & -(xZ)^{-1} \mathcal{V}_d \, (\Lambda_d^{(2)})^{1/2} \\ x^k M^\dagger \mathcal{W}_d \, (\Lambda_d^{(0)})^{-1/2} & (x^k M)^{-1} \mathcal{V}_d \, (\Lambda_d^{(2)})^{1/2} \end{pmatrix}, \tag{4.24}$$

respectively. Here $\mathcal{W}_d$ and $\mathcal{V}_d$ are 3×3 unitary matrices which are determined such that the matrices

$$\mathcal{W}_d^{-1} (A_d + B_d) \mathcal{W}_d = \Lambda_d^{(0)}, \qquad \mathcal{V}_d^{-1} (A_d^{-1} + B_d^{-1})^{-1} \mathcal{V}_d = \Lambda_d^{(2)} \tag{4.25}$$

become diagonal. As a consequence we can expect to have a nontrivial CKM matrix

$$V^{CKM} = \mathcal{V}_u^{-1} \, \mathcal{V}_d. \tag{4.26}$$

Note that $\mathcal{V}_u$ is determined such that $\mathcal{V}_u^{-1} B_d \mathcal{V}_u$ is diagonal. If the relation

$$|\det(A_d + B_d)| \simeq |\det A_d| \gg |\det B_d| \tag{4.27}$$

is satisfied, the mixing is small and we have

$$(A_d^{-1} + B_d^{-1})^{-1} \simeq B_d. \tag{4.28}$$

This implies that mass pattern of down-type quarks is the same as that of up-type quarks and that $\mathcal{V}_d \simeq \mathcal{V}_u$. In this case V^{CKM} becomes almost a unit matrix.

To get a phenomenologically viable solution, large mixings between D'^c and D^c are preferable. Thus we impose the maximal mixing in which $(A_d^{-1})_{ij}$ and $(B_d^{-1})_{ij}$ are the same order. The maximal mixing is realized under the condition

$$2 z_{33} = k - 1 - \alpha - \gamma + \beta + \delta. \tag{4.29}$$

Note that k is an odd integer. Under the above condition on z_{33} the eigenvalues of $A_d + B_d$ become

$$O(x^{2(k+\alpha+\gamma+\delta)}), \qquad O(x^{2(k+\beta+\delta-\alpha)}), \qquad O(x^{2k}). \tag{4.30}$$

It follows that extra down-type heavy quarks have their masses

$$M_S\, x^{k+\alpha+\gamma+\delta}, \qquad M_S\, x^{k+\beta+\delta-\alpha}, \qquad M_S\, x^k. \tag{4.31}$$

Main components of these eigenstates are D_1-$(O(1)D_1^c+O(1)D_2^c)$, D_2-$D_3'^c$ and D_3-$D_2'^c$, respectively. On the other hand, down-type light quarks have their masses

$$v_d\, x^{\alpha+\beta+\gamma+\delta}, \qquad v_d\, x^{\beta+\gamma+\delta}, \qquad v_d\, x^{-\alpha+\beta+\delta}, \tag{4.32}$$

which correspond to observed d-, s- and b-quarks. These eigenstates are approximately D_1-$(O(1)D_1^c + O(1)D_2^c)$, D_2-$D_1'^c$ and D_3-$D_3'^c$, respectively. It should be noted that we have very large D_i^c-$D_i'^c$ mixings. The unitary matrix $\mathcal{V}_d$ which diagonalizes $A_d^{-1} + B_d^{-1}$, is expressed as

$$\mathcal{V}_d = \begin{pmatrix} 1-O(x^{2\alpha}) & O(x^\alpha) & O(x^{\alpha+\gamma}) \\ O(x^\alpha) & 1-O(x^{2\alpha}) & O(x^\gamma) \\ O(x^{\alpha+\gamma}) & O(x^\gamma) & 1-O(x^{2\gamma}) \end{pmatrix}. \tag{4.33}$$

Corresponding elements of the matrices $\mathcal{V}_u$ and $\mathcal{V}_d$ are in the same order of magnitudes but their coefficients of the leading term in off-diagonal elements are different with each other because of the maximal mixing. Consequently, the CKM matrix is given by

$$V^{CKM} = \mathcal{V}_u^{-1}\mathcal{V}_d = \begin{pmatrix} 1-O(x^{2\alpha}) & O(x^\alpha) & O(x^{\alpha+\gamma}) \\ O(x^\alpha) & 1-O(x^{2\alpha}) & O(x^\gamma) \\ O(x^{\alpha+\gamma}) & O(x^\gamma) & 1-O(x^{2\gamma}) \end{pmatrix}. \tag{4.34}$$

It is worth noting that large D_i^c-$D_i'^c$ mixings play an essential role in generating a nontrivial CKM matrix. An early attempt of explaining the CKM matrix via D_i^c-$D_i'^c$ mixings has been made in Ref.[15], in which a SUSY $SO(10)$ model was taken.

Confronting the CKM matrix obtained here with the observed one, it is feasible for us to take

$$\alpha = 1.0 \times w, \qquad \gamma = 2.0 \times w \tag{4.35}$$

with $x^w = \lambda = \sin\theta_C$, where θ_C is the Cabbibo angle. In this parametrization we get

$$V^{CKM} = \begin{pmatrix} 1-O(\lambda^2) & O(\lambda) & O(\lambda^3) \\ O(\lambda) & 1-O(\lambda^2) & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1-O(\lambda^4) \end{pmatrix}. \tag{4.36}$$

Further, if we set

$$\beta = 2.5 \times w, \qquad \delta = 1.5 \times w, \tag{4.37}$$

then we have quark masses

$$m_u \;=\; O(\lambda^7 v_u), \qquad m_c = O(\lambda^{3.5} v_u), \qquad m_t = O(v_u), \tag{4.38}$$

$$m_d \;=\; O(\lambda^7 v_d), \qquad m_s = O(\lambda^6 v_d), \qquad m_b = O(\lambda^3 v_d). \tag{4.39}$$

These results are in line with the observed values. Since α, β, γ, and δ are set to be even positive integers, w should be a multiple of 4 in this case. Taking $x^w = \lambda \sim 0.22$ and $x^{2sk-2} = m_{3/2}/M_S = 10^{-(15\sim16)}$ into account, we obtain the constraint

$$sk = (10 \sim 14) \times w. \tag{4.40}$$

As a typical example, we will often refer the set of parameters

$$\begin{aligned} &s = 10, \qquad k = 5, \qquad w = 4, \qquad e = 0, \\ &\alpha = 4, \qquad \beta = 10, \qquad \gamma = 8, \qquad \delta = 6, \end{aligned} \tag{4.41}$$

in which we have $K = 51$. In this case we have $x \simeq 0.7$ and $x^k \simeq 0.15$. Consequently, when $M_S = 10^{18}$GeV, the symmetry breaking scales $\langle S_0 \rangle$ and $\langle N_0^c \rangle$ turn out to be $\sim 7 \times 10^{17}$GeV and $\sim 1.5 \times 10^{17}$GeV, respectively. Since the symmetry breaking scales are very large, we have the standard model gauge group over the wide energy range.

5 Spectra of leptons

Let us now study the mass matrices for lepton sector, in which L-H_d mixing occurs at energies below the scale $\langle N_0^c \rangle$. Colorless $SU(2)_L$-doublet fields L and H_d are not distinguished with each other under G_{st}. Then, hereafter we denote R-parity odd H_{di} as L'_i $(i = 1, 2, 3)$. As mentioned in section 2, H_{ui} and L'_i $(i = 1, 2, 3)$ in chiral multiplets do not develop their VEVs. It follows that there exist no mixings of $SU(2)_L \times U(1)_Y$ gauge superfields with H_{ui} and L'_i ($i = 1, 2, 3$). Since both L and L' are $SU(2)_L$-doublets, the CKM matrix for lepton sector becomes a unit matrix irrespective of the magnitude of L-L' mixing. For charged leptons the superpotential is

$$W_E \sim (S_0\overline{S})^{h_{ij}} S_0 L'_i H_{uj} + (S_0\overline{S})^{m_{ij}} L_i (N_0^c H_{uj} + H_{d0} E_j^c), \tag{5.1}$$

where the exponents h_{ij} are integers in the range $0 \leq h_{ij} < K$ and satisfy

$$2h_{ij} + b_i + b_j + a_0 + 2 \equiv 0 \qquad \text{mod } K. \tag{5.2}$$

Thus we have

$$2h_{ij} \equiv 2h_{33} + \left(\begin{array}{ccc} 2\beta + 2\delta & \beta + 2\delta & \beta + \delta \\ \beta + 2\delta & 2\delta & \delta \\ \beta + \delta & \delta & 0 \end{array} \right)_{ij} \qquad \text{mod } K \tag{5.3}$$

with $2h_{33} \equiv -2b_3 - a_0 - 2$. As before, we introduce a 3×3 matrix H with elements

$$H_{ij} = O(x^{2h_{ij}}). \tag{5.4}$$

The mass matrix for charged leptons has the form

$$\widehat{M}_l = \begin{array}{c} \\ L'^- \\ L^- \end{array} \begin{array}{c} \begin{array}{cc} \;\; H_u^+ & \;\; E^{c+} \end{array} \\ \left(\begin{array}{cc} xH & 0 \\ x^k M & \rho_d M \end{array} \right) \end{array} \tag{5.5}$$

in M_S units. This $\widehat{M}_l$ is also a 6×6 matrix and can be diagonalized by a bi-unitary transformation as

$$\widehat{\mathcal{V}}_l^{-1} \widehat{M}_l \widehat{\mathcal{U}}_l. \tag{5.6}$$

From Eqs.(5.5) and (5.6) the matrix

$$\widehat{\mathcal{U}}_l^{-1} \widehat{M}_l^\dagger \widehat{M}_l \widehat{\mathcal{U}}_l = \widehat{\mathcal{U}}_l^{-1} \left(\begin{array}{cc} A_l + B_l & \epsilon_d B_l \\ \epsilon_d B_l & \epsilon_d^2 B_l \end{array} \right) \widehat{\mathcal{U}}_l \tag{5.7}$$

is diagonal, where

$$A_l = x^2 H^\dagger H, \qquad B_l = x^{2k} M^\dagger M. \tag{5.8}$$

The analysis is parallel to that of down-type quark masses in the previous section. We have the eigen equation

$$\det\left(A_l + B_l - \frac{\eta}{M_S^2}\right) = 0 \tag{5.9}$$

for heavy states. For three light states their masses squared are given by the eigen equation

$$\det\left(x^{-2k}(A_l^{-1} + B_l^{-1})^{-1} - \frac{\eta}{v_d^2}\right) = 0. \tag{5.10}$$

The light states correspond to observed charged leptons. Due to L-L' mixings mass pattern of charged leptons could be changed from that of up-type quarks. Introducing appropriate unitary matrices $\mathcal{W}_l$ and $\mathcal{V}_l$, we can diagonalize $(A_l + B_l)$ and $(A_l^{-1} + B_l^{-1})^{-1}$ as

$$\mathcal{W}_l^{-1}(A_l + B_l)\mathcal{W}_l = \Lambda_l^{(0)}, \qquad \mathcal{V}_l^{-1}(A_l^{-1} + B_l^{-1})^{-1}\mathcal{V}_l = \Lambda_l^{(2)}, \tag{5.11}$$

where $\Lambda_l^{(0)}$ and $\Lambda_l^{(2)}$ are diagonal 3×3 matrices. Masses of charged leptons are written as

$$m_{li}^2 = v_d^2 \left(x^{-2k}\Lambda_l^{(2)}\right)_{ii} \qquad (i = 1, 2, 3). \tag{5.12}$$

Explicit forms of $\widehat{\mathcal{V}}_l$ and $\widehat{\mathcal{U}}_l$ are

$$\widehat{\mathcal{V}}_l \simeq \begin{pmatrix} xH\mathcal{W}_l\,(\Lambda_l^{(0)})^{-1/2} & -(xH^\dagger)^{-1}\mathcal{V}_l\,(\Lambda_l^{(2)})^{1/2} \\ x^k M\mathcal{W}_l\,(\Lambda_l^{(0)})^{-1/2} & (x^k M^\dagger)^{-1}\mathcal{V}_l\,(\Lambda_l^{(2)})^{1/2} \end{pmatrix}, \tag{5.13}$$

$$\widehat{\mathcal{U}}_l \simeq \begin{pmatrix} \mathcal{W}_l & -\epsilon_d(A_l + B_l)^{-1}B_l\mathcal{V}_l \\ \epsilon_d B_l(A_l + B_l)^{-1}\mathcal{W}_l & \mathcal{V}_l \end{pmatrix}. \tag{5.14}$$

In the same way as the case of down-type quarks, we also choose a large mixing solution. Defining an even integer ξ by

$$\xi = 2h_{33} - k + 1 - (\alpha + \gamma - \beta - \delta), \tag{5.15}$$

we now impose the condition

$$\alpha + \gamma + \xi - \beta - \delta \sim 0. \tag{5.16}$$

This condition means that

$$2h_{33} \sim k - 1. \tag{5.17}$$

In this case eigenvalues of $A_l + B_l$ become $x^{2(k+\beta+2\delta)}$, $x^{2(k+\delta)}$ and x^{2k}. Thus masses of three heavy charged leptons are

$$M_S\, x^{k+\beta+2\delta}, \qquad M_S\, x^{k+\delta}, \qquad M_S\, x^k, \tag{5.18}$$

whose eigenstates are mainly L_1-H_{u1}, L'_1-H_{u2} and $(O(1)L'_2 + O(1)L'_3)$-H_{u3}, respectively. On the other hand, light charged leptons have their masses of

$$v_d\, x^{\alpha+\beta+\gamma+\delta+\xi}, \qquad v_d\, x^{\alpha+\gamma+\delta}, \qquad v_d\, x^{\gamma}, \tag{5.19}$$

which correspond to observed e-, μ- and τ-leptons, respectively. Main components of these eigenstates are L_2-E^c_1, L_3-E^c_2 and $(O(1)L'_2 + O(1)L'_3)$-E^c_3, respectively. The unitary matrix $\mathcal{V}_l$ which diagonalizes $A_l^{-1} + B_l^{-1}$, is expressed as

$$\mathcal{V}_l = \begin{pmatrix} 1 - O(x^{2\beta}) & O(x^{\beta}) & O(x^{\beta+\delta}) \\ O(x^{\beta}) & 1 - O(x^{2\delta}) & O(x^{\delta}) \\ O(x^{\beta+\delta}) & O(x^{\delta}) & 1 - O(x^{2\delta}) \end{pmatrix}. \tag{5.20}$$

Taking

$$\xi = w \tag{5.21}$$

together with the above parametrization (4.35) and (4.37), we obtain

$$m_e = O(\lambda^8 v_d), \qquad m_\mu = O(\lambda^{4.5} v_d), \qquad m_\tau = O(\lambda^2 v_d) \tag{5.22}$$

and

$$\mathcal{V}_l = \begin{pmatrix} 1 - O(\lambda^5) & O(\lambda^{2.5}) & O(\lambda^4) \\ O(\lambda^{2.5}) & 1 - O(\lambda^3) & O(\lambda^{1.5}) \\ O(\lambda^4) & O(\lambda^{1.5}) & 1 - O(\lambda^3) \end{pmatrix}. \tag{5.23}$$

We now proceed to study mass pattern of neutral sector. In the present framework there are fifteen neutral fields, i.e., H_{ui}^0, $L_i'^0$, L_i^0, N_i^c and S_i ($i = 1, 2, 3$). For neutral fields the superpotential is of the form

$$\begin{aligned} W_N \ \sim \ & (S_0\overline{S})^{h_{ij}} S_0 L_i' H_{uj} + (S_0\overline{S})^{m_{ij}} L_i (N_0^c H_{uj} + H_{u0} N_j^c) \\ & + (S_0\overline{S})^{s_{ij}} (S_i\overline{S})(S_j\overline{S}) + (S_0\overline{S})^{t_{ij}} (S_i\overline{S})(N_j^c\overline{N}^c) \\ & + (S_0\overline{S})^{n_{ij}} (N_i^c\overline{N}^c)(N_j^c\overline{N}^c), \end{aligned} \tag{5.24}$$

where the exponents s_{ij}, t_{ij} and n_{ij} are determined by

$$\begin{aligned} & 2s_{ij} + a_i + a_j + 2\overline{a} + 2 \equiv 0, \\ & 2t_{ij} + a_i + b_j + \overline{a} + \overline{b} + 2 \equiv 0, \qquad \mod K \\ & 2n_{ij} + b_i + b_j + 2\overline{b} + 2 \equiv 0 \end{aligned} \tag{5.25}$$

in the range $0 \leq s_{ij}, t_{ij}, n_{ij} < K$. From Eqs.(4.2), (4.14) and (5.2) these equations are put into the form

$$\begin{aligned} & 2s_{ij} \equiv 2z_{ij} + a_0 - 2\overline{a}, \\ & 2t_{ij} \equiv 2m_{ij} + b_0 - \overline{a} - \overline{b}, \qquad \mod K \\ & 2n_{ij} \equiv 2h_{ij} + a_0 - 2\overline{b}. \end{aligned} \tag{5.26}$$

In the above we have imposed the ansatzs

$$2m_{33} \ \equiv \ 0, \tag{5.27}$$

$$2z_{33} \ \equiv \ k - 1 - \alpha - \gamma + \beta + \delta, \tag{5.28}$$

$$2h_{33} \ \equiv \ k - 1 + \alpha + \gamma - \beta - \delta + \xi \sim k - 1 \tag{5.29}$$

together with the conditions

$$a_0 + \overline{a} \equiv 2, \qquad b_0 + \overline{b} \equiv 2k, \qquad a_0 + 2b_0 \equiv 2e, \qquad e = 0, 1. \tag{5.30}$$

Therefore, the exponents s_{ij}, t_{ij} and n_{ij} are rewritten as

$$\begin{aligned}
2s_{ij} &\equiv 2z_{ij} - 2 + 2e - 2k - \xi, \\
2t_{ij} &\equiv 2m_{ij} - 2 + 2e - 2k, \qquad\qquad \mathrm{mod}\ K \\
2n_{ij} &\equiv 2h_{ij} + 2e - 4k.
\end{aligned} \tag{5.31}$$

Introducing 3×3 matrices S, T and N with elements

$$S_{ij} = O(x^{2s_{ij}}), \quad T_{ij} = O(x^{2t_{ij}}), \quad N_{ij} = O(x^{2n_{ij}}), \tag{5.32}$$

we have a 15×15 mass matrix

$$\widehat{M}_N = \begin{array}{c} \\ H_u^0 \\ L'^0 \\ L^0 \\ N^c \\ S \end{array} \begin{array}{c} \begin{array}{ccccc} H_u^0 & L'^0 & L^0 & N^c & S \end{array} \\ \begin{pmatrix} 0 & xH & x^k M^T & 0 & 0 \\ xH & 0 & 0 & 0 & 0 \\ x^k M & 0 & 0 & \rho_u M & 0 \\ 0 & 0 & \rho_u M^T & x^{2k} N & x^{k+1} T^T \\ 0 & 0 & 0 & x^{k+1} T & x^2 S \end{pmatrix} \end{array} \tag{5.33}$$

in M_S units for neutral sector, where $\rho_u = v_u / M_S$. Since $SU(2)_L$ symmetry is preserved above the electroweak scale, the eigen equation for six heavy states is the same as Eq.(5.9). For another nine states we have an approximate eigen equation

$$\det \left(\widehat{M}_{LNS} - \frac{\eta}{M_S} \right) = 0, \tag{5.34}$$

where $\widehat{M}_{LNS}$ is defined by

$$\widehat{M}_{LNS} = \begin{pmatrix} 0 & \epsilon_u (\Lambda_l^{(2)})^{1/2} \mathcal{V}_l^{-1} & 0 \\ \epsilon_u \mathcal{V}_l (\Lambda_l^{(2)})^{1/2} & x^{2k} N & x^{k+1} T^T \\ 0 & x^{k+1} T & x^2 S \end{pmatrix} \tag{5.35}$$

with $\epsilon_u = x^{-k} \rho_u$. Light neutrino masses are given by

$$m_{\nu i} = \frac{m_{li}^2}{M_S\, x^{2k}} \left(\frac{v_u}{v_d} \right)^2 \left(\mathcal{V}_l^{-1} \Delta_N \mathcal{V}_l \right)_{ii} \tag{5.36}$$

with

$$\Delta_N = (N - T^T S^{-1} T)^{-1}. \tag{5.37}$$

This type of mass matrix has been discussed in Ref.[16]. Let us suppose that $\widehat{M}_{LNS}$ is a 3×3 matrix. When $N - TS^{-1}T \sim N$ in order of magnitude, the usual seesaw mechanism [17] is at work. On the other hand, when $N < TS^{-1}T$, another type of seesaw mechanism takes place. In the present framework $\widehat{M}_{LNS}$ is a 9×9 matrix. The exponents $2n_{ij}$ in N_{ij} are equal to those in $(T^T S^{-1} T)_{ij}$ in modulus K. The relative magnitude of N_{ij} to $(T^T S^{-1} T)_{ij}$ depends on the value of k. In the case $k > \frac{1}{2}(\alpha+\gamma) - 1 + e$ which corresponds to small s ($s \lesssim 8$), light neutrino masses are not so small. For instance, we have $m_{\nu\tau} = O(1\mathrm{keV})$. So we do not adopt this case. In the case $k \leq \frac{1}{2}(\alpha+\gamma) - 1 + e$ which corresponds to $s \gtrsim 8$, light neutrino masses are extremely small. Specifically, the latter case yields

$$\left(\mathcal{V}_l^{-1} \Delta_N \mathcal{V}_l\right)_{ii} = \{x^{3k+1-2(\beta+\delta+e)}, \quad x^{3k+1-2(\delta+e)}, \quad x^{3k+1-2e}\}_i. \tag{5.38}$$

Combining this with the result (5.19) for m_{li}, we have the light neutrino masses

$$m_{\nu i} = \frac{v_u^2}{M_S} x^{k+1-2e} \times \{x^{2(\alpha+\gamma+\xi)}, \quad x^{2(\alpha+\gamma)}, \quad x^{2\gamma}\}. \tag{5.39}$$

The previous parametrization (4.35) and (4.37) leads us to

$$m_{\nu i} = \frac{v_u^2}{M_S} x^{k+1-2e} \times \{\lambda^8, \quad \lambda^6, \quad \lambda^4\}. \tag{5.40}$$

It follows that

$$\frac{m_{\nu e}}{\lambda^4} \sim \frac{m_{\nu\mu}}{\lambda^2} \sim m_{\nu\tau} \leq O(10^{-7}\mathrm{eV}). \tag{5.41}$$

The calculated neutrino masses seem to be too small. According to the analyses of solar neutrino [18], atmospheric neutrino[19] and cosmological constraints[20], it is preferable that three typical mass scales of neutrinos are $\sim 10^{-3}$eV, $\sim 10^{-1}$eV and ~ 10eV. The ratios $m_{\nu e}/m_{\nu\mu}$ and $m_{\nu\mu}/m_{\nu\tau}$ obtained here are consistent with those among the above three typical mass scales. As pointed out at the beginning of this section, the CKM-matrix for lepton sector is a unit matrix irrespective of the magnitude of L-L' mixing. The situation is unchanged even through seesaw mechanism. This is because the fields N^c and S are $SU(2)_L$-singlet. In addition, the components of N^c and S in light neutrinos are very small. Thus we have no flavor-changing charged currents at tree level. Recently, by introducing discrete symmetries which yield appropriate texture zeros in Yukawa couplings, the hierarchical pattern of neutrino masses has been examined in Ref.[21]. Finally, we touch upon the remaining eigenvalues of Eq.(5.34). Three pairs of heavy states which are $G_{st}-$neutral have their masses of

$$M_S x^{-k-1+2e} \times \{x^{\beta+\delta-\alpha-\xi}, \quad x^{\alpha+\gamma}, \quad x^{\alpha+\gamma+\delta+\xi}\}. \tag{5.42}$$

6 Proton decay

After studying the particle spectra of down-type colored fields in Φ_0 and $\overline{\Phi}$, we explore the proton stability in this section. Under the discrete R-symmetry the superpotential of down-type colored fields is of the form

$$\begin{aligned} W_g \sim\ & (S_0\overline{S})^q S_0 g_0 g_0^c + (S_0\overline{S})^{sk-4-q}(\overline{S}\overline{g}\,\overline{g}^c + (S_0\overline{S})^{1-e} N_0^c \overline{S} D_0^c \overline{g}^c) \\ & + (S_0\overline{S})^{sk-1}(g_0\overline{g} + g_0^c\overline{g}^c + (S_0\overline{S})^{1-e} g_0 N_0^c D_0^c) + (S_0\overline{S})^{(s-2)k+e-1}\overline{g}\,\overline{N}^c\overline{D}^c \\ & + (S_0\overline{S})^{(s-1)k} D_0^c\overline{D}^c + (S_0\overline{S})^{(s-2)k+q+e+1} S_0\overline{N}^c g_0^c \overline{D}^c, \end{aligned} \tag{6.1}$$

where $q = k + \frac{1}{2}\xi - e - 2$. When S_0, $\overline{S}$, N_0^c and $\overline{N}^c$ develop non-zero VEVs, the superpotential induces the mass matrix

$$M_g = \begin{array}{c} \\ g_0 \\ \overline{D}^c \\ \overline{g}^c \end{array} \begin{array}{c} \begin{array}{ccc} \quad g_0^c \quad & \quad \overline{g} \quad & \quad D_0^c \quad \end{array} \\ \left(\begin{array}{ccc} O(x^{2k-3-2e+\xi}) & O(\rho) & O(\rho\, x^{k+2-2e}) \\ O(\rho\, x^{1-k+\xi}) & O(\rho\, x^{-3k+2e}) & O(\rho\, x^{-2k+2}) \\ O(\rho) & O(\rho\, x^{-2k-1+2e-\xi}) & O(\rho\, x^{-k+1-\xi}) \end{array} \right) \end{array} \tag{6.2}$$

in M_S units, where $\rho = m_{3/2}/M_S = 10^{-(15\sim16)}$. This mass matrix is diagonalized by a bi-unitary transformation as

$$\mathcal{V}_g^{-1} M_g\, \mathcal{U}_g \tag{6.3}$$

with

$$\mathcal{V}_g = \begin{pmatrix} 1 & O(\rho\, x^{-3k+4+2e}) & O(\rho\, x^{-2k+3+2e-\xi}) \\ O(\rho\, x^{-3k+4+2e}) & 1 & O(x^{k-1-\xi}) \\ O(\rho\, x^{-2k+3+2e-\xi}) & O(x^{k-1-\xi}) & 1 \end{pmatrix}, \tag{6.4}$$

$$\mathcal{U}_g = \begin{pmatrix} 1 & O(\rho\, x^{-2k+3+2e}) & O(\rho\, x^{-k+5-\xi}) \\ O(\rho\, x^{-2k+3+2e}) & 1 & O(x^{k+2-2e}) \\ O(\rho\, x^{-k+5-\xi}) & O(x^{k+2-2e}) & 1 \end{pmatrix}. \tag{6.5}$$

The eigenvalues are given by

$$\begin{aligned} M_{gA} &= O(M_S x^{2k-3-2e+\xi}), \\ M_{gB} &= O(m_{3/2} x^{-3k+2e}), \\ M_{gC} &= O(m_{3/2} x^{-k+1-\xi}). \end{aligned} \tag{6.6}$$

From explicit forms of $\mathcal{V}_g$ and $\mathcal{U}_g$ we find that three eigenstates g_A-g_A^c, g_B-g_B^c and g_C-g_C^c are approximately g_0-g_0^c, $\overline{D}^c$-$\overline{g}$ and $\overline{g}^c$-D_0^c states, respectively. In the typical example (4.41) M_{gA} is nearly GUT-scale ($\sim 10^{16}$GeV). By contrast, M_{gB} and M_{gC} are as small as $O(10^{5\sim6}$GeV). Then, at first sight, it seems that dimension-five operators mediated by these rather light colored fields lead to fast proton decay. However, this is not the case. The dimension-five operators mediated by light colored fields are strongly suppressed because of extremely small effective couplings. This is due to the fact that (1,2), (2,1), (1,3) and (3,1) entries of $\mathcal{V}_g$ are $O(10^{-13})$. In what follows we explain this situation more explicitly.

Since R-parity of quark and lepton superfields are odd, those of $SU(2)_L$-singlet colored superfields mediating proton decay should be even. The relevant superfields are g_0, g_0^c, and D_0^c which reside in $\Phi_0(\mathbf{27})$. Effective trilinear couplings with g_0, g_0^c and D_0^c are given by

$$\begin{aligned} W_g^{eff} \sim\ & (Q^T Z Q + N^{cT} H D^c + E^{cT} H U^c) g_0 + (Q^T Z L + U^{cT} H D^c) g_0^c \\ & + (Q^T M L' + D'^T M N^c + D'^{cT} M U^c) D_0^c, \end{aligned} \tag{6.7}$$

where 3×3 matrices Z, H and M have already been determined in sections 4 and 5. Superfields g_0, g_0^c and D_0^c are expressed in terms of mass eigenstates as

$$\begin{aligned} g_0 &= \mathcal{V}_{g11}\, g_A + \mathcal{V}_{g12}\, g_B + \mathcal{V}_{g13}\, g_C, \\ g_0^c &= \mathcal{U}_{g11}\, g_A^c + \mathcal{U}_{g12}\, g_B^c + \mathcal{U}_{g13}\, g_C^c, \\ D_0^c &= \mathcal{U}_{g31}\, g_A^c + \mathcal{U}_{g32}\, g_B^c + \mathcal{U}_{g33}\, g_C^c. \end{aligned} \tag{6.8}$$

Taking $\mathcal{V}_{g11},\ \mathcal{U}_{g11} \simeq 1$ and $\mathcal{U}_{g31} \sim \rho x^{-k+5-\xi}$ into account, we can obtain dominant dimension-five operators from g_A-g_A^c exchange

$$\frac{1}{M_S} x^{-2k+3+2e-\xi}(Q^T Z Q + N^{cT} H D^c + E^{cT} H U^c)(Q^T Z L + U^{cT} H D^c). \tag{6.9}$$

Similarly, dominant dimension-five operators from g_C-g_C^c exchange are

$$\frac{1}{M_S} x^{-k+2+2e}(Q^T Z Q + N^{cT} H D^c + E^{cT} H U^c)(Q^T M L' + D'^T M N^c + D'^{cT} M U^c). \tag{6.10}$$

The prefactor is induced from $\mathcal{V}_{g13}(M_{gC})^{-1}\mathcal{U}_{g33}$. In dimension-five operators for g_B-g_B^c exchange the prefactor is $\mathcal{V}_{g12}(M_{g_B})^{-1}\mathcal{U}_{g32}$, which is smaller than the one for g_C-g_C^c exchange by the factor $x^{2k+4(1-e)}$. Therefore, the study of dimension-five operators coming from g_A-g_A^c and g_C-g_C^c exchanges suffices to explore the proton stability.

We now rewrite the above operators in terms of quark and lepton mass eigenstates, which are represented by using the symbol "tilde". In order to implement this rewriting, we can use the transfer

$$\begin{aligned} Q &\to \mathcal{V}_u \tilde{Q}' = \begin{pmatrix} \mathcal{V}_u & 0 \\ 0 & \mathcal{V}_u \end{pmatrix} \begin{pmatrix} \tilde{U} \\ V^{CKM} \tilde{D} \end{pmatrix}, \\ U^c &\to \mathcal{U}_u \tilde{U}^c, \qquad D^c \to \widehat{\mathcal{U}}_{d22} \tilde{D}^c, \\ D' &\to \mathcal{V}_d \tilde{D}, \qquad D'^c \to \widehat{\mathcal{U}}_{d12} \tilde{D}^c, \\ L &\to \widehat{\mathcal{V}}_{l22} \tilde{L}, \qquad L' \to \widehat{\mathcal{V}}_{l12} \tilde{L}, \\ E^c &\to \mathcal{V}_l \tilde{E}^c, \end{aligned} \tag{6.11}$$

where

$$\widehat{\mathcal{U}}_{d12} = -(xZ)^{-1}\mathcal{V}_d(\Lambda_d^{(2)})^{1/2}, \tag{6.12}$$

$$\widehat{\mathcal{U}}_{d22} = (x^k M)^{-1}\mathcal{V}_d(\Lambda_d^{(2)})^{1/2}, \tag{6.13}$$

$$\widehat{\mathcal{V}}_{l12} = -(xH^\dagger)^{-1}\mathcal{V}_l(\Lambda_l^{(2)})^{1/2}, \tag{6.14}$$

$$\widehat{\mathcal{V}}_{l22} = (x^k M^\dagger)^{-1}\mathcal{V}_l(\Lambda_l^{(2)})^{1/2}, \tag{6.15}$$

which are three-by-three blocks of matrices given in Eqs.(4.24) and (5.13). Light component of N^c is extremely small and then its contribution to nucleon decay is negligible. Therefore, Eq. (6.9) is translated into

$$\begin{aligned}\frac{1}{M_S}x^{-2k+3+2e-\xi}[\tilde{Q}'^T(\mathcal{V}_u^T Z\mathcal{V}_u)\tilde{Q}' \times \tilde{Q}'^T(\mathcal{V}_u^T Z\widehat{\mathcal{V}}_{l22})\tilde{L} \\ +\ \tilde{E}^{cT}(\mathcal{V}_l^T H\mathcal{U}_u)\tilde{U}^c \times \tilde{U}^{cT}(\mathcal{U}_u^T H\widehat{\mathcal{U}}_{d22})\tilde{D}^c].\end{aligned} \tag{6.16}$$

Similarly, Eq. (6.10) is put into

$$\begin{aligned}\frac{1}{M_S}x^{-k+2+2e}[\tilde{Q}'^T(\mathcal{V}_u^T Z\mathcal{V}_u)\tilde{Q}' \times \tilde{Q}'^T(\mathcal{V}_u^T M\widehat{\mathcal{V}}_{l12})\tilde{L} \\ +\ \tilde{E}^{cT}(\mathcal{V}_l^T H\mathcal{U}_u)\tilde{U}^c \times \tilde{U}^{cT}(\mathcal{U}_u^T M^T\widehat{\mathcal{U}}_{d12})\tilde{D}^c].\end{aligned} \tag{6.17}$$

The dimension-five operators result in nucleon decay via gaugino- or Higgsino-dressing processes [22][23]. Among various dressing processes the exchange of charged wino or Higgsino give predominant contributions to nucleon decay. Since $SU(2)_L$-singlet states $\tilde{U^c}$, $\tilde{D^c}$, $\tilde{E^c}$ do not couple to $SU(2)_L$-gauginos, dominant dimension-five operators with charged wino-dressing processes turn out to be the first terms in Eqs.(6.16) and (6.17). Thus we have dominant operators incorporating charged wino-dressing processes

$$\begin{aligned}\frac{1}{M_S}x^{-2k+3+2e-\xi}\tilde{Q}_1'(\mathcal{V}_u^T Z\mathcal{V}_u)_{11}\tilde{Q}_1' \times \tilde{Q}_2'(\mathcal{V}_u^T Z\widehat{\mathcal{V}}_{l22})_{2j}\tilde{L}_j \\ +\frac{1}{M_S}x^{-k+2+2e}\tilde{Q}_1'(\mathcal{V}_u^T Z\mathcal{V}_u)_{11}\tilde{Q}_1' \times \tilde{Q}_2'(\mathcal{V}_u^T M\widehat{\mathcal{V}}_{l12})_{2j}\tilde{L}_j.\end{aligned} \tag{6.18}$$

Simple calculations yield

$$
\begin{aligned}
(\mathcal{V}_u^T Z \mathcal{V}_u)_{ij} &= O(Z_{ij}), && (6.19)\\
(\mathcal{V}_u^T Z \widehat{\mathcal{V}}_{l22})_{ij} &= O(x^{k-1+\xi})(\mathcal{V}_u^T M \widehat{\mathcal{V}}_{l12})_{ij} \\
&= x^{k-1+\beta+\delta} \times \begin{pmatrix} O(x^{\alpha+\gamma+\xi}) & O(x^{\alpha+\gamma}) & O(x^{\gamma}) \\ O(x^{\gamma+\xi}) & O(x^{\gamma}) & O(x^{\gamma-\alpha}) \\ O(x^{\xi}) & O(1) & O(x^{-\alpha}) \end{pmatrix}_{ij}. && (6.20)
\end{aligned}
$$

From these relations Eq.(6.18) becomes

$$
\frac{1}{M_S}(\tilde{Q}'_1 \tilde{Q}'_1 \tilde{Q}'_2 \tilde{L}_j)\, x^{2(\alpha+\beta+\gamma+\delta)+1+2e-\xi} \times (x^{-\alpha+\xi},\ x^{-\alpha},\ x^{-2\alpha})_j. \tag{6.21}
$$

This implies that a dominant mode of proton decay is $p \to K^0\ + \mu^+$. In this decay mode the magnitude of the dimension-five operator is given by

$$
\frac{1}{M_S} x^{\alpha+2(\beta+\gamma+\delta)+1+2e-\xi} \ \simeq\ \frac{1}{M_S}\lambda^{12} \ \gtrsim\ 10^{-(25.5\sim 26.5)}\mathrm{GeV}^{-1}. \tag{6.22}
$$

The second terms in Eqs.(6.16) and (6.17) contribute to nucleon decay via charged Higgsino-dressing processes. The relevant terms are

$$
\begin{aligned}
&\frac{1}{M_S} x^{-2k+3+2e-\xi} \tilde{E}_i^c (\mathcal{V}_l^T H \mathcal{U}_u)_{ij} \tilde{U}_j^c \times \tilde{U}_m^c (\mathcal{U}_u^T H \widehat{\mathcal{U}}_{d22})_{m1} \tilde{D}_1^c \\
&\qquad + \frac{1}{M_S} x^{-k+2+2e} \tilde{E}_i^c (\mathcal{V}_l^T H \mathcal{U}_u)_{ij} \tilde{U}_j^c \times \tilde{U}_m^c (\mathcal{U}_u^T M \widehat{\mathcal{U}}_{d12})_{m1} \tilde{D}_1^c, && (6.23)
\end{aligned}
$$

where $(j,\, m) = (1,\, 2),\ (2,\, 1)$. Using the relations

$$
\begin{aligned}
(\mathcal{V}_l^T H \mathcal{U}_u)_{ij} &= O(H_{ij}), && (6.24)\\
(\mathcal{U}_u^T H \widehat{\mathcal{U}}_{d22})_{ij} &= O(x^{k-1})(\mathcal{U}_u^T M \widehat{\mathcal{U}}_{d12})_{ij} \\
&= x^{k-1+\gamma} \times \begin{pmatrix} O(x^{\alpha+\beta+\delta}) & O(x^{\alpha+\beta+\delta}) & O(x^{\beta+\delta}) \\ O(x^{\alpha+\delta}) & O(x^{\alpha+\delta}) & O(x^{\delta}) \\ O(x^{\alpha}) & O(x^{\alpha}) & O(1) \end{pmatrix}_{ij}, && (6.25)
\end{aligned}
$$

we obtain the dimension-five operators for $SU(2)_L$-singlet fields

$$
\frac{1}{M_S}(\tilde{E}_j^c \tilde{U}_2^c \tilde{U}_1^c \tilde{D}_1^c)\, x^{\beta+2(\alpha+\gamma+\delta)+1+2e} \times (1,\ x^{-\beta},\ x^{-\beta-\delta})_j. \tag{6.26}
$$

In the Higgsino-dressing processes the operators are multiplied by their Yukawa couplings. As a consequence we have a dominant contribution in the case $j = 3$. The above operator multiplied by the Yukawa couplings for $\tilde{E}^c_3$ and $\tilde{U}^c_2$ becomes

$$\frac{1}{M_S} x^{2(\alpha+2\gamma+\delta)+1+2e} \simeq \frac{1}{M_S} \lambda^{13}, \tag{6.27}$$

in magnitude and results in the decay $p \to K^+ + \overline{\nu}_\tau$.

In conclusion of this section, the main mode of proton decay is $p \to K^0 + \mu^+$, in which the magnitude of the dimension-five operator is about $\lambda^{12}/M_S \simeq 10^{-(25.5\sim 26.5)}\text{Gev}^{-1}$. This implies that the proton lifetime is about $10^{33\sim 35}$yr. This result is consistent with the present experimental data.

7 Gauge coupling unification

As is well-known, there is a discrepancy between the string scale M_S and the MSSM unification scale $\sim 2 \times 10^{16}$GeV. Main concern here is whether or not we can reconcile this discrepancy in the present model. For this purpose we study the renormalization group evolution of the gauge couplings in the model up to two-loop order.

In the preceding sections particle spectra have been already studied. Unlike the MSSM, in the present model there are many extra intermediate-scale fields, which are tabulated in Table II. In particular, the contributions of $(H_{u0},\ H_{d0},\ \overline{H}_u,\ \overline{H}_d)$ and $(\overline{g},\ \overline{g}^c,\ \overline{D}^c,\ D^c_0)$ are significant, because their masses are lying in rather low energy region ($10^{2\sim 7}$GeV).

Table II

The evolution equations for $\alpha_i = g_i^2/4\pi$ are generally given up to two-loop order by

$$\mu^2 \frac{d\alpha_i}{d\mu^2} = \frac{1}{4\pi}\left[b_i + \sum_j \frac{b_{ij}}{4\pi}\alpha_j - \frac{a_i}{4\pi} \right] \alpha_i^2, \tag{7.1}$$

where μ is the running mass scale [24]. The coefficients b_i, b_{ij} and a_i are determined by the particle content of the model. The third term in the r.h.s. represents the contribution of Yukawa couplings. In the present calculation we take accout only of the largest Yukawa couplings $f = M_{33}$, namely, Yukawa couplings of the third generation $\Phi_3(\mathbf{15},\, \mathbf{1})\Phi_3(\mathbf{6}^*,\, \mathbf{2})\Phi_0(\mathbf{6}^*,\, \mathbf{2})$ and for simplicity we neglect the renormalization group evolution of the Yukawa couplings. In our analysis it is assumed that string threshold corrections are negligibly small.

In the region between M_S and $\langle S_0 \rangle = M_S\, x$, where the gauge symmetry is $SU(6)\times SU(2)_R$, we have

$$b_i = \begin{pmatrix} -8 \\ 9 \end{pmatrix}, \qquad b_{ij} = \begin{pmatrix} 9 & 15 \\ 175 & 81 \end{pmatrix}, \qquad \frac{a_i}{y} = \begin{pmatrix} 28 \\ 60 \end{pmatrix}, \tag{7.2}$$

where $y = f^2/4\pi$ is taken to be a constant. In the region between $M_S\, x$ and $\langle N_0^c \rangle = M_S\, x^k$, where the gauge symmetry is $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, we get

$$b_i = \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}, \qquad b_{ij} = \begin{pmatrix} 118 & 9 & 15 \\ 45 & 53 & 15 \\ 75 & 15 & 81 \end{pmatrix}, \qquad \frac{a_i}{y} = \begin{pmatrix} 36 \\ 32 + 4n(S_3) \\ 56 + 4n(S_3) \end{pmatrix} \tag{7.3}$$

with

$$n(S_3) \;=\; \begin{cases} 1 & M_S\, x > \mu \geq M_S\, x^{-k+\gamma+2e-1} \\ 0 & M_S\, x^{-k+\gamma+2e-1} > \mu \geq M_S\, x^k. \end{cases} \tag{7.4}$$

In the wide energy region ranging from $M_S\, x^k$ to $m_{3/2} = M_S\, x^{2sk-2}$ the gauge group coincides with the standard model gauge group. From Table II we can calculate the coefficients, which are of the forms

$$b_i \;=\; \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 2/5 \end{pmatrix} n_g + \begin{pmatrix} 0 \\ 1 \\ 3/5 \end{pmatrix} n_H, \tag{7.5}$$

$$
\begin{aligned}
b_{ij} &= \begin{pmatrix} 14 & 9 & 11/5 \\ 24 & 18 & 6/5 \\ 88/5 & 18/5 & 38/5 \end{pmatrix} + \begin{pmatrix} 34/3 & 0 & 4/15 \\ 0 & 0 & 0 \\ 32/15 & 0 & 8/75 \end{pmatrix} n_g \\
&\quad + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 7 & 3/5 \\ 0 & 9/5 & 9/25 \end{pmatrix} n_H, \qquad (7.6) \\
\frac{a_i}{y} &= \begin{pmatrix} 4 \\ 8 \\ 44/5 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ 14/5 \end{pmatrix} n(D_3^c) + \begin{pmatrix} 8 \\ 3 \\ 31/5 \end{pmatrix} n(D_0^c) \\
&\quad + \begin{pmatrix} 0 \\ 2 \\ 6/5 \end{pmatrix} n(N_3^c). \qquad (7.7)
\end{aligned}
$$

In these expressions n_H and n_g stand for the numbers of doublet Higgses and extra down-type colored fields, respectively and are given by

$$
n_H = \begin{cases} 4 & M_S\, x^k > \mu \geq M_S\, x^{k+\delta} \\ 3 & M_S\, x^{k+\delta} > \mu \geq M_S\, x^{k+\beta+2\delta} \\ 2 & M_S\, x^{k+\beta+2\delta} > \mu \geq M_S\, x^{2sk-4k+2e-1} \\ 1 & M_S\, x^{2sk-4k+2e-1} > \mu \geq M_S\, x^{2sk-2}, \end{cases} \qquad (7.8)
$$

$$
n_g = \begin{cases} 5 & M_S\, x^k > \mu \geq M_S\, x^{2k-3-2e+\xi} \\ 4 & M_S\, x^{2k-3-2e+\xi} > \mu \geq M_S\, x^{k+\beta+\delta-\alpha} \\ 3 & M_S\, x^{k+\beta+\delta-\alpha} > \mu \geq M_S\, x^{k+\alpha+\gamma+\delta} \\ 2 & M_S\, x^{k+\alpha+\gamma+\delta} > \mu \geq M_S\, x^{2sk-3k+2e-2} \\ 1 & M_S\, x^{2sk-3k+2e-2} > \mu \geq M_S\, x^{2sk-k-1-\xi} \\ 0 & M_S\, x^{2sk-k-1-\xi} > \mu \geq M_S\, x^{2sk-2}. \end{cases} \qquad (7.9)
$$

$n(D_3^c)$, $n(D_0^c)$ and $n(N_3^c)$ are

$$
n(D_3^c) = \begin{cases} 1 & M_S\, x^k > \mu \geq M_S\, x^{k+\beta+\delta-\alpha} \\ 0 & M_S\, x^{k+\beta+\delta-\alpha} > \mu \geq M_S\, x^{2sk-2}, \end{cases} \qquad (7.10)
$$

$$
n(D_0^c) = \begin{cases} 1 & M_S\, x^k > \mu \geq M_S\, x^{2sk-k-1+\xi} \\ 0 & M_S\, x^{2sk-k-1+\xi} > \mu \geq M_S\, x^{2sk-2}, \end{cases} \qquad (7.11)
$$

$$
n(N_3^c) = \begin{cases} 1 & M_S\, x^k > \mu \geq M_S\, x^{\alpha+\gamma-k+2e-1} \\ 0 & M_S\, x^{\alpha+\gamma-k+2e-1} > \mu \geq M_S\, x^{2sk-2}. \end{cases} \qquad (7.12)
$$

It should be noted that in the present model we obtain $n_H - n_g = 0$ over rather wide energy range. By contrast, in the MSSM we have $n_H - n_g = 1$. In the region between $m_{3/2}$ and M_Z where supersymmetry is broken, all superparticles except for light Higgses do not contribute the evolution equations. This leads us to

$$b_i = \begin{pmatrix} -7 \\ -7/3 \\ 23/5 \end{pmatrix}, \qquad b_{ij} = \begin{pmatrix} -26 & 9/2 & 11/10 \\ 12 & 97/6 & 3/2 \\ 44/5 & 9/2 & 217/50 \end{pmatrix}, \qquad \frac{a_i}{y} = \begin{pmatrix} 2 \\ 2 \\ 16/5 \end{pmatrix}. \tag{7.13}$$

We are now in a position to solve the evolution equation numerically. The behavior of the renormalization group flow is shown in Fig.1, in which we choose a typical example (4.41). In the present calculation the parameters are taken as

$$M_S = 0.5 \times 10^{18}\mathrm{GeV}, \quad m_{3/2} = 200\mathrm{GeV}, \quad \alpha_{string}^{-1} = 14.0, \quad f = 1.7, \tag{7.14}$$

where α_{string} represents the unified gauge coupling at the string scale. Resulting values of $\alpha_i^{-1}(M_Z)$ are

$$\alpha_1^{-1}(M_Z) = 58.96, \quad \alpha_2^{-1}(M_Z) = 26.03, \quad \alpha_3^{-1}(M_Z) = 8.66. \tag{7.15}$$

Compared with the present experimental values [25]

$$\alpha_1^{-1}(M_Z) = 58.95 \pm 0.08, \quad \alpha_2^{-1}(M_Z) = 29.66 \pm 0.07, \quad \alpha_3^{-1}(M_Z) = 8.48 \pm 0.43, \tag{7.16}$$

the calculated $\alpha_1^{-1}(M_Z)$ and $\alpha_3^{-1}(M_Z)$ are consistent with the data, while the calculated $\alpha_2^{-1}(M_Z)$ is smaller than the observed one by ~ 3.5. Consequently, our analysis shows that the gauge coupling unification is not achieved at the string scale. This suggests that we are not successful in getting proper particle spectra of extra intermediate-scale matter.

Fig. 1

8 Summary

In the context of level-one string model we have explored a possibility that characteristic pattern of quark-lepton masses and the CKM matrix have their origin in the discrete R-symmetry and mixing mechanism. In this paper we have chosen $Z_K \times Z_2$ symmetry with $K = sk + 1$ as an example of the discrete R-symmetry. The Z_2-symmetry is assumed so as to be in accord with the R-parity in the MSSM and is unbroken down to the electroweak scale. The vector-like multiplets Φ_0, $\overline{\Phi}$ and the chiral multiplets Φ_i $(i = 1, 2, 3)$ are assigned to even and odd R-parity, respectively. Under this assignment no mixing occurs between the vector-like multiplets and the chiral multiplets. The Z_K symmetry is used as a horizontal symmetry. The Z_K symmetry controls a large hierarchy of the energy scales of the symmetry breaking and particle spectra. Triplet-doublet splitting problem and the μ-problem are solved as a result of the discrete symmetry. The assignment of Z_K-charges to chiral multiplets is of great importance in explaining the observed hierarchical pattern of quark-lepton masses.

The mass hierarchy of up-type quarks is a direct result of the horizontal discrete symmetry. On the other hand, for down-type quarks there appears a mixing between D^c and $D'^c (= g^c)$. Due to the maximal mixing mass pattern of down-type quarks is different from that of up-type quarks. The mass hierarchy obtained here is

$$m_u = O(v_u\, x^{\alpha+\beta+\gamma+\delta}), \quad m_c = O(v_u\, x^{\gamma+\delta}), \quad m_t = O(v_u), \tag{8.1}$$

$$m_d = O(v_d\, x^{\alpha+\beta+\gamma+\delta}), \quad m_s = O(v_d\, x^{\beta+\gamma+\delta}), \quad m_b = O(v_d\, x^{-\alpha+\beta+\delta}). \tag{8.2}$$

These results are consistent with observations under the parametrization $\alpha = w$, $\beta = 2.5w$, $\gamma = 2w$, $\delta = 1.5w$ and $x^w = \lambda \sim 0.22$. Further we obtain a phenomenologically viable CKM matrix. In lepton sector L-$L' (= H_d)$ mixing occurs. Hence, mass pattern of charged leptons is also changed from that of up-type quarks under a large mixing.

The obtained mass hierachy for leptons is

$$m_e = O(v_d\, x^{\alpha+\beta+\gamma+\delta+\xi}), \quad m_\mu = O(v_d\, x^{\alpha+\gamma+\delta}), \quad m_\tau = O(v_d\, x^{\gamma}). \tag{8.3}$$

The CKM matrix in lepton sector amounts to a unit matrix irrespectively of the magnitude of L-$L'(= H_d)$ mixing. This is because both L and $L'(= H_d)$ are $SU(2)_L$-doublets. Therefore, lepton flavor violating processes are extremely suppressed. See-saw mechanism is at work for neutrinos. For large s ($s \geq 8$) light neutrino masses are

$$\begin{aligned} m_{\nu e} &= \frac{v_u^2}{M_S} O(x^{k+1-2e+2(\alpha+\gamma+\xi)}), \\ m_{\nu\mu} &= \frac{v_u^2}{M_S} O(x^{k+1-2e+2(\alpha+\gamma)}), \\ m_{\nu\tau} &= \frac{v_u^2}{M_S} O(x^{k+1-2e+2\gamma}). \end{aligned} \tag{8.4}$$

These masses seem to be too small compared with those expected from solar neutrino and atmospheric neutrino data. In the present framework the proton lifetime is $10^{33\sim35}$yr, which is long enough to be consistent with experimental data. The suppression of the dimension-five operators occurs because of the superheavy mass of the mediating particle for certain processes and because of the extremely small couplings for the other processes. On the other hand, we are not succesfull in achieving the unification of gauge couplings at the string scale. Nevertheless, it is suggestive that the obtained numerical value $\alpha_{string}^{-1} \sim 14$ corresponds nearly to the self-dual point $g_{string} = 1$ with respect to S-duality (strong/weak duality).

Both in D^c-$D'^c(= g^c)$ and L-$L'(= H_d)$ mixings the mass differences between heavy states and light states are extremely large in order of magnitudes. This implies that these mixings do not practically bring about flavor-changing neutral current processes. In addition, flavor-changing neutral current processes via superparticle exchanges at loop level are also suppressed enough to be consistent with experimental data, provided that the soft SUSY breaking parameters are universal at the string scale. More explicitly, the most stringent experimental bound on the mass difference of squarks $\tilde{d}$ and $\tilde{s}$ is derived from the K^0-$\bar{K}^0$ mixing. As pointed out in section 4, $SU(2)_L$-singlet components of down-type quarks d, s and b are nearly $O(1)D_1^c + O(1)D_2^c$, $D_1'^c$ and $D_3'^c$, respectively. Although D^c and D'^c are indistinguishable from each other under the standard model gauge group, D^c and D'^c reside in $(\mathbf{6}^*, \mathbf{2})$ and $(\mathbf{15}, \mathbf{1})$ of $SU(6) \times SU(2)_R$, respectively. Further, D^c and D'^c reside in $(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ and $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ of $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, respectively. Therefore, gauge interactions cause soft SUSY breaking masses of $\tilde{d}_R^c$ and $\tilde{s}_R^c$ to evolve differently through radiative corrections in the energy region ranging from M_S to $M_S\, x^k$. However, in the present model this energy range is rather nallow. In fact, x^k is about $10^{-0.8}$ in a typical example. Consequently, it can be shown that the difference $\delta m^2 = m^2(\tilde{d}_R^c) - m^2(\tilde{s}_R^c)$ remains small at low energies. Let us estimate numerically the difference δm^2 in a typical example. When we assume $\delta m^2(M_S) = 0$, the difference at the scale $M_S\, x$ $(= \langle S_0 \rangle)$ becomes

$$\delta m^2(M_S\, x) \simeq -0.016 \times M_A^2 \tag{8.5}$$

through the RG evolution, where M_A is an averaged gaugino mass. Subsequently, the RG evolution from $M_S\, x$ to $M_S\, x^k$ $(= \langle N_0^c \rangle)$ leads to

$$\delta m^2(M_S\, x^k) - \delta m^2(M_S\, x) \simeq 0.008 \times M_A^2. \tag{8.6}$$

Combining these two results, we obtain

$$\delta m^2(M_S\, x^k) \simeq -0.008 \times M_A^2. \tag{8.7}$$

Since Yukawa couplings of down-type quarks are tiny in case of $\tan\beta \sim 1$, the contributions of Yukawa interactions to $\delta m^2(m_{3/2})$ are small compared with $\delta m^2(M_S\, x^k)$. It follows that $\delta m^2(m_{3/2}) \simeq \delta m^2(M_S\, x^k)$, which is consistent with a bound on $\delta m^2(m_{3/2})$ given in Ref.[26].

Although we did not deal with CP-violation, there are two possibilities of introducing the CP-phase in the present framework. One possibility is that the CP-phase comes from complex VEVs of moduli fields. In this case the coefficients of the terms in the string-scale superpotential are complex in general. Another possibility is the case that the coefficients in the superpotential are all real but VEVs $\langle S_0 \rangle$, $\langle \overline{S} \rangle$, $\langle N_0^c \rangle$ and $\langle \overline{N}^c \rangle$, are complex. When we take the relative phase of $\langle N_0^c \rangle \langle \overline{N}^c \rangle$ to $\langle S_0 \rangle \langle \overline{S} \rangle$ into account, there appears CP-violating phase in the model.

Appendix A

In this appendix we show that the minimization of the scalar potential yields tree-level breaking of the gauge symmetry under an appropriate condition on soft SUSY breaking parameters. In minimal supergravity model the soft SUSY breaking terms are given by [27]

$$\begin{aligned}\mathcal{L}_{soft} &= \int d^4\theta\, \Phi^\dagger \left[m_{3/2}\, \theta^2 B + m_{3/2}\, \overline{\theta}^2 B^* - m_{3/2}^2\, \theta^2\overline{\theta}^2 C\right] \exp(2gV)\Phi \\ &\qquad - \left[\int d^2\theta\, m_{3/2}\, \theta^2 A\, W + \text{h. c.}\right]. \qquad \text{(A.1)}\end{aligned}$$

Here $m_{3/2}$ is supposed to be $O(1\text{TeV})$. The universal soft SUSY breaking parameters A, B and C are generally zero or order unity. Although A and B are generally complex numbers, C is a real one. This type of $\mathcal{L}_{soft}$ leads to the scalar potential

$$\begin{aligned}V &= \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + m_{3/2}(A\, W + A^*\, W^*) \\ &\quad + m_{3/2} \sum_i \Big(B\, \phi_i \frac{\partial W}{\partial \phi_i} + B^*\, \phi_i^* \frac{\partial W^*}{\partial \phi_i^*}\Big) \\ &\quad + m_{3/2}^2 (C + |B|^2) \sum_i |\phi_i|^2 + (D\text{−term}), \qquad \text{(A.2)}\end{aligned}$$

where ϕ_i is a scalar component of the chiral superfield Φ_i. In the above expression it is assumed that the terms of higher powers of $1/M_{pl}$ are negligibly small. We will shortly show that this assumption is justified.

For illustration, we take one set of vector-like multiplet Φ and $\overline{\Phi}$, whose scalar components are denoted as ϕ and $\overline{\phi}$, respectively. Let us consider the case that the nonrenormalizable interaction

$$W = \lambda M_S^{3-2n} (\Phi\overline{\Phi})^n \qquad \text{(A.3)}$$

is compatible with the discrete symmetry, where λ is a positive $O(1)$ constant and n is a large positive integer. In a typical example (4.41) we put $n = sk = 50$. For simplicity we denote dimensinless quantities V/M_S^4 and ϕ/M_S by the same letters as the original V and ϕ. Thus

$$\begin{aligned} V &= n^2\lambda^2 \left\{ \left|\phi^{n-1}\overline{\phi}^n\right|^2 + \left|\phi^n\overline{\phi}^{n-1}\right|^2 \right\} \\ &\quad +\rho\lambda \left\{ (A+2nB)(\phi\overline{\phi})^n + (A^*+2nB^*)(\phi^*\overline{\phi}^*)^n \right\} \\ &\quad +\rho^2(C+|B|^2)(|\phi|^2+|\overline{\phi}|^2) + (D\text{−term}) \end{aligned} \tag{A.4}$$

with $\rho = m_{3/2}/M_S$. Minimization of V leads to the D-flat direction

$$|\langle\phi\rangle| = |\langle\overline{\phi}\rangle| = x. \tag{A.5}$$

Writing the phase factor of VEVs explicitly as

$$\langle\phi\rangle\langle\overline{\phi}\rangle = x^2 e^{i\theta}, \tag{A.6}$$

we have the scalar potential

$$V = 2n^2\lambda^2 x^{4n-2} + 2\rho\,|A+2nB|\,\lambda x^{2n}\cos\delta + 2\rho^2(C+|B|^2)x^2, \tag{A.7}$$

where $\delta = n\theta + \arg(A+2nB)$. From the stationary condition $\partial V/\partial\delta = 0$ and $x \geq 0$ the phase θ is determined as $\cos\delta = -1$. Therefore, the dependence of V on x is given by

$$V = 2\left[n\lambda x^{2n-1} - \rho\left|B+\frac{A}{2n}\right|x\right]^2 - 2\rho^2\left[\left|B+\frac{A}{2n}\right|^2 - C - |B|^2\right]x^2. \tag{A.8}$$

Consequently, if the inequality

$$\left|B+\frac{A}{2n}\right|^2 > C + |B|^2 \tag{A.9}$$

holds, V is minimized at a nonzero value of x, namely at $x \sim \rho^{1/(2n-2)}$. If $C \leq 0$, the above inequality is satisfied, for example, in the case $|\arg(A\,B^*)| < \pi/2$ even for large n. It is worth emphasizing that the soft SUSY breaking mass parameter

$(C+|B|^2)$ is not necessarily negative. If only the above inequality is satisfied, the gauge symmtry is spontaneously broken at tree level. It is not necessary for us to rely on the radiative symmetry breaking mechanism. In this paper the exponent n is taken to be rather large. The larger n implies the larger VEV $|\langle\phi\rangle| = M_S x$. The large value of $|\langle\phi\rangle|$ is consistent with the tree-level symmtry breaking.

In supergravity theory with canonical Kähler potential the supersymmetric term of the scalar potential is expressed as

$$V = e^{K/M^2}\left[\sum_i \left|\frac{\partial W}{\partial\phi_i} + \frac{\phi_i^*}{M^2}W\right|^2 - \frac{3}{M^2}|W|^2\right] + (D\text{−term}) \qquad \text{(A.10)}$$

with $M = M_{pl}/\sqrt{8\pi} \gtrsim M_S$. In the present model we get

$$\begin{aligned} \left|\langle\frac{\partial W}{\partial\phi}\rangle\right| &\simeq n\lambda x^{2n-1}M_S^2, \\ \left|\langle\frac{\phi^*}{M^2}W\rangle\right| &\simeq \lambda x^{2n+1}(\frac{M_S}{M})^2 M_S^2, \\ \left|\langle\frac{1}{M}W\rangle\right| &\simeq \lambda x^{2n}(\frac{M_S}{M})M_S^2. \end{aligned} \qquad \text{(A.11)}$$

Since n is large and $x < 1$, $M_S/M \lesssim 1$, V is dominated by $\partial W/\partial\phi$. The overall factor $\langle\exp(K/M^2)\rangle$ is order unity. Therefore, the above analysis is relevant to the issue of the symmetry breaking.

Appendix B

In this appendix we address to the issue of R-parity conservation within the present framework. It is shown that if eigenvalues of the mass matrix

$$\widehat{M}_{NS} = \begin{pmatrix} x^{2k}N & x^{k+1}T^T \\ x^{k+1}T^T & x^2 S \end{pmatrix} \tag{B.1}$$

in M_S units are sufficiently large compared with $m_{3/2}$, the scalar potential is minimized along the direction where R-parity is conserved. The mass matrix $\widehat{M}_{NS}$ is a submatrix of $\widehat{M}_{LNS}$ given in section 5 and yields masses of R-parity odd and G_{st}-neutral superfields. It has already been found in section 5 that the above condition is satisfied for the solutions discussed in the text.

The superpotential can be separated as

$$W = W_1 + W_2, \tag{B.2}$$

where W_1 is a function only of R-parity even fields S_0, $\overline{S}$, N_0^c and $\overline{N}^c$, while each term of W_2 contains R-parity odd fields $\Phi_i = S_j,\ N_j^c$ $(i = 1, \cdots, 6\,;\ j = 1,\ 2,\ 3)$. In the same manner as the notations in appendix A, we now use dimensionless quatities in M_S units. Due to the $Z_K \times Z_2$ symmetry the explicit form of W_1 is given by

$$W_1 = \sum_{r=0}^{s} c_r (S_0\overline{S})^{(s-r)k} (N_0^c \overline{N}^c)^r, \tag{B.3}$$

where c_r are $O(1)$ constants in M_S units. This superpotential satisfies a relation

$$W_1 = \frac{1}{2sk}\left[S_0\frac{\partial W_1}{\partial S_0} + \overline{S}\frac{\partial W_1}{\partial \overline{S}}\right] + \frac{1}{2s}\left[N_0^c\frac{\partial W_1}{\partial N_0^c} + \overline{N}^c\frac{\partial W_1}{\partial \overline{N}^c}\right]. \tag{B.4}$$

W_2 is a even function of Φ_i. Consequently, the scalar potential is of the form

$$V = V_1 + V_2 \tag{B.5}$$

with

$$V_1 \;=\; \left|\frac{\partial W}{\partial S_0} + \rho(B + \frac{A}{2sk})^* S_0^*\right|^2 + \left|(S_0 \to \overline{S})\right|^2$$

$$
\begin{aligned}
& + \left| \frac{\partial W}{\partial N_0^c} + \rho (B + \frac{A}{2s})^* N_0^{c*} \right|^2 + \left| (N_0^c \to \overline{N}^c) \right|^2 \\
& - \rho^2 \left(\left| B + \frac{A}{2sk} \right|^2 - C - |B|^2 \right) \left(|S_0|^2 + |\overline{S}|^2 \right) \\
& - \rho^2 \left(\left| B + \frac{A}{2s} \right|^2 - C - |B|^2 \right) \left(|N_0^c|^2 + |\overline{N}^c|^2 \right),
\end{aligned} \tag{B.6}
$$

$$
V_2 = \sum_{i=1}^{6} \left| \frac{\partial W_2}{\partial \phi_i} + \rho B^* \phi_i^* \right|^2 + \rho (A \tilde{W}_2 + A^* \tilde{W}_2{}^*) + \rho^2 C \sum_{i=1}^{6} |\phi_i|^2. \tag{B.7}
$$

Here ϕ_i's ($i = 1, \cdots, 6$) represent scalar components of S_j and N_j^c ($j = 1, 2, 3$) and $\tilde{W}_2$ is defined by

$$
\tilde{W}_2 = W_2 - \frac{1}{2sk} \left[S_0 \frac{\partial W_2}{\partial S_0} + \overline{S} \frac{\partial W_2}{\partial \overline{S}} \right] - \frac{1}{2s} \left[N_0^c \frac{\partial W_2}{\partial N_0^c} + \overline{N}^c \frac{\partial W_2}{\partial \overline{N}^c} \right]. \tag{B.8}
$$

Scalar components of S_0, $\overline{S}$, N_0^c and $\overline{N}^c$ are denoted by the same letters as the superfields themselves. As discussed in appendix A, under the assumption

$$
\left| B + \frac{A}{2sk} \right|^2, \ \left| B + \frac{A}{2s} \right|^2 > C + |B|^2, \tag{B.9}
$$

S_0, $\overline{S}$, N_0^c and $\overline{N}^c$ develop nonzero VEVs and then the gauge symmetry is spontaneously broken at tree level. The stationary condition is satisfied at nonzero values of S_0, $\overline{S}$, N_0^c and $\overline{N}^c$ and vanishing $\langle \phi_i \rangle$. At this stationary point we get a negative value of the scalar potential

$$
V = V_1 = -O(\rho^2 \langle S_0 \rangle^2). \tag{B.10}
$$

The question here is whether this point is the absolute minimum or not.

Let us suppose that some of ϕ_i develop nonzero VEVs at the absolute minimum point. For such ϕ_i, if

$$
\left| \langle \frac{\partial W_2}{\partial \phi_i} \rangle \right| \gg \rho |\langle \phi_i \rangle|, \tag{B.11}
$$

then V_2 is dominated as

$$
V_2 \simeq \sum_i \left| \langle \frac{\partial W_2}{\partial \phi_i} \rangle \right|^2 \gg \rho^2 \sum_i |\langle \phi_i \rangle|^2 \tag{B.12}
$$

and lifts up the scalar potential V. It follows that this point can not be the absolute minimun. Therefore, the relation

$$\left| \langle \frac{\partial W_2}{\partial \phi_i} \rangle \right| \lesssim \rho |\langle \phi_i \rangle| \tag{B.13}$$

should be satisfied for all i. On the other hand, the mass matrix of $\phi_i(\Phi_i)$ is given by

$$\langle \frac{\partial^2 W_2}{\partial \phi_i \partial \phi_j} \rangle = \left(\widehat{M}_{NS} \right)_{ij} . \tag{B.14}$$

This matrix yields masses of R-parity odd and G_{st}-neutral superfields, which are assumed to be sufficiently larger than $\rho = m_{3/2}/M_S$. Namely, when we introduce a unitary matrix $\widehat{U}_{NS}$ which diagonalizes $\widehat{M}_{NS}$, this assumption is expressed as

$$\sum_{j,k} \left(\widehat{U}_{NS}^{-1} \right)_{ij} \langle \frac{\partial^2 W_2}{\partial \phi_j \partial \phi_k} \rangle \left(\widehat{U}_{NS} \right)_{ki} \gg \rho \tag{B.15}$$

for all i. Although we have six unknown parameters $\langle \phi_i \rangle$, there are twelve constraints (B.13) and (B.15) on $\langle \phi_i \rangle$ in which the orders of magnitude are quite different. Since we have too much constraints on $\langle \phi_i \rangle$, in generic case there are no consistent solutions except for $\langle \phi_i \rangle = 0$ for all i. Consequently the absolute minimum of V is achieved at $\langle \phi_i \rangle = 0$. This means that R-parity is conserved.

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Table I

	$\overline{\Phi}$	Φ_0	Φ_1	Φ_2	Φ_3
$(\mathbf{15},\mathbf{1})$	$(\overline{a},+)$	$(a_0,+)$	$(a_1,-)$	$(a_2,-)$	$(a_3,-)$
$(\mathbf{6}^*,\mathbf{2})$	$(\overline{b},+)$	$(b_0,+)$	$(b_1,-)$	$(b_2,-)$	$(b_3,-)$

Table II

R-parity	Matter fields	X (mass scale: $m = O(M_S\, x^X)$)
$+$	Q_0, L_0, $\overline{Q}$, $\overline{L}$, $\frac{1}{\sqrt{2}}(S_0 - \overline{S})$	1
	U_0^c, E_0^c, $\overline{U}^c$, $\overline{E}^c$, $\frac{1}{\sqrt{2}}(N_0^c - \overline{N}^c)$	k
	$\frac{1}{\sqrt{2}}(S_0 + \overline{S})$	$2sk - 2$
	$\frac{1}{\sqrt{2}}(N_0^c + \overline{N}^c)$	$2sk - 2k$
	H_{u0}, H_{d0}, $\overline{H}_u$, $\overline{H}_d$,	$2sk - 4k + 2e - 1$, $2sk - 2e + 1$
	g_0, g_0^c, D_0^c, $\overline{g}$, $\overline{g}^c$, $\overline{D}^c$	$2k - 2e - 3$, $2sk - 3k + 2e - 2$, $2sk - k - 1 - \xi$
$-$	H_{ui}, $\tilde{L}'_i$	$k + \beta + 2\delta$, $k + \delta$, k
	D'_i, $\tilde{D}'^c_i$	$k + \alpha + \gamma + \delta$, $k + \beta + \delta - \alpha$, k
	N_i^c, S_i	$-k + \gamma + 2e - 1$, $-k + \alpha + \gamma + 2e - 1$, $-k + \beta + 2\delta + 2e - 1$
	Q_i, U_i^c, $\tilde{D}_i^c$, $\tilde{L}_i$, E_i^c	$> 2sk - 2$

Table Captions

Table I The numbers a_i and b_i $(i = 0, 1, 2, 3)$ in the parentheses represent the Z_K-charges of chiral superfields $\Phi(\mathbf{15}, \mathbf{1})$ and $\Phi(\mathbf{6}^*, \mathbf{2})$, respectively. $\overline{a}$ and $\overline{b}$ stand for those of mirror chiral superfields $\overline{\Phi}(\mathbf{15}^*, \mathbf{1})$ and $\overline{\Phi}(\mathbf{6}, \mathbf{2})$, respectively. Respective Z_2-charges (R-parity) of the superfields are also listed.

Table II Particle spectra in the present model. The number X stands for the exponent of x for the mass scale $m = O(M_S\, x^X)$ of each superfield. Note that $x^{2sk-2} = m_{3/2}/M_S$ and $K = sk + 1$. The parameters α, β, γ and δ are given in section 4. In this table $\tilde{D}_i^c$ and $\tilde{D}_i'^c$ ($\tilde{L}_i$ and $\tilde{L}_i'$) stand for light and heavy eigenstates, respectively, which are derived via mixings between D_i^c and $D_i'^c$ (L_i and L_i').

Figure Captions

Fig.1 The renormalization group flow of gauge couplings. The string scale, the soft SUSY breaking scale and the unified gauge coupling are taken as $M_S = 0.5 \times 10^{18}$GeV, $m_{3/2} = 200$GeV and $\alpha_{string}^{-1} = 14.0$, respectively. The Yukawa coupling of the third generation is fixed to $f = 1.7$.