Quark Mixings in $SU(6) \times SU(2)_R$ and Suppression of V_{ub}

M. Matsuda[1][*] and T. Matsuoka[2][†]

[1]*Department of Physics and Astronomy, Aichi University of Education,*

Kariya, Aichi, 448-8542 Japan

[2]*Kogakkan University, Nabari, Mie, 518-0498 Japan*

Abstract

The quark mixing matrix V_{CKM} is studied in depth on the basis of superstring inspired $SU(6) \times SU(2)_R$ model with global flavor symmetries. The sizable mixings between right-handed down-type quark D^c and colored Higgs field g^c potentially occur but no such mixings in up-type quark sector. In the model the hierarchical pattern of V_{CKM} is understood systematically. It is shown that due to large D^c-g^c mixings V_{ub} is naturally suppressed compared to V_{td}. It is pointed out that the observed suppression of V_{ub} is in favor of the presence of $SU(2)_R$ gauge symmetry but not in accord with generic $SU(5)$ GUT.

---

[*]E-mail address:mmatsuda@auecc.aichi-edu.ac.jp

[†]E-mail address:matsuoka@kogakkan-u.ac.jp

Fermion masses and mixings are closely related to each other. Indeed, there have appeared many attempts to express the elements of Cabibbo-Kobayashi-Maskawa mixing matrix V_{CKM} in terms of quark masses[1]. The observed fermion masses have the hierarchical pattern though tiny mass pattern of neutrinos is still unclear. The V_{CKM} also has peculiar pattern. It seems that these characteristic patterns shed some lights on the gauge symmetry and matter contents at the unification scale. The purpose of this work is to understand the following two challenging issues and then to explore the gauge symmetry and matter contents at the unification scale.

(i). If we take a naive viewpoint of GUT, it is plausible that up- and down-type quarks reside in the same irreducible representation of GUT gauge group. This implies that the CKM matrix should be unit matrix in contrast to the experimental facts, which show nonzero values for off-diagonal elements[2]. On the other hand, up- and down-type quarks have distinct hierarchical mass pattern each other($m_u/m_d < m_c/m_s < m_t/m_b$). If Yukawa couplings of up- and down-quark sectors are independent each other, it is natural that the V_{CKM} would have large off-diagonal elements. This is also inconsistent with experimental facts which show that V_{CKM} might be almost unit matrix. How can we understand this property of the CKM matrix ?

(ii). Among the characteristic pattern of the CKM matrix, the asymmetric feature of the matrix is noticeable. Specifically, the element V_{ub} in V_{CKM} is rather small compared to V_{td}, i.e.,

$$
\begin{aligned}
|V_{td}| &\simeq |V_{cd} \cdot V_{ts}| \simeq \lambda^3, \qquad (\lambda \simeq 0.22) \\
|V_{ub}| &\simeq \lambda |V_{cb} \cdot V_{us}| \simeq \lambda^4,
\end{aligned} \tag{1}
$$

where the second relation is suggested by the experimental results $|V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$[2]. How can we understand this feature of the CKM matrix ?

In the context of the string inspired $SU(6) \times SU(2)_R$ model with global flavor symmetries, it has been shown that the main pattern of fermion masses and mixings can be understood as a consequence of mixings between quarks(leptons) and extra particles[3, 4, 5]. Along the previous works we investigate the structure of V_{CKM} and quark masses in depth. In this paper we show that the above features can be naturally understood in the $SU(6) \times SU(2)_R$ model.

The model discussed here is the same as in Ref.[3, 4, 5]. In this study we choose $SU(6) \times SU(2)_R$ as the unification gauge symmetry at the string scale M_S, which can be derived from the perturbative heterotic superstring theory via the flux breaking[6]. In terms of E_6 we set matter superfields which consist of three family and one vector-like multiplet, i.e.,

$$
3 \times \mathbf{27}(\Phi_{1,2,3}) + (\mathbf{27}(\Phi_0) + \overline{\mathbf{27}}(\overline{\Phi})). \tag{2}
$$

Under $G = SU(6) \times SU(2)_R$, the superfields Φ in **27** of E_6 are decomposed into two groups as

$$\Phi(\mathbf{27}) = \begin{cases} \phi(\mathbf{15}, \mathbf{1}) & : & Q, L, g, g^c, S, \\ \psi(\overline{\mathbf{6}}, \mathbf{2}) & : & (U^c, D^c), (N^c, E^c), (H_u, H_d), \end{cases} \tag{3}$$

where g, g^c and H_u, H_d represent colored Higgs and doublet Higgs fields, respectively. Under G, doublet Higgs and color-triplet Higgs fields belong to different representations and this situation is favorable to solve the triplet-doublet splitting problem. N^c is the right-handed neutrino superfield and S is an $SO(10)$-singlet. Although D^c and $g^c(L$ and $H_d)$ have the same quantum numbers under the standard model gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, they belong to different irreducible representations of G. We assign odd R-parity for $\Phi_{1,2,3}$ and even for Φ_0 and $\overline{\Phi}$, respectively. Since ordinary Higgs doublets have even R-parity, they belong to Φ_0. It is assumed that R-parity remains unbroken down to the electroweak scale.

The gauge symmetry G is spontaneously broken in two steps at the scale $\langle S_0 \rangle = \langle \overline{S} \rangle$ and $\langle N_0^c \rangle = \langle \overline{N}^c \rangle$ as

$$G = SU(6) \times SU(2)_R \xrightarrow{\langle S_0 \rangle} SU(4)_{\mathrm{PS}} \times SU(2)_L \times SU(2)_R \xrightarrow{\langle N_0^c \rangle} G_{SM}, \tag{4}$$

where $SU(4)_{\mathrm{PS}}$ represents the Pati-Salam $SU(4)$[7]. Hereafter it is supposed that the symmetry breaking scales are roughly $\langle S_0 \rangle = 10^{17 \sim 18}$GeV and $\langle N_0^c \rangle = 10^{15 \sim 17}$GeV. Gauge invariant trilinear couplings in the superpotential W are of the forms

$$(\phi(\mathbf{15}, \mathbf{1}))^3 = QQg + Qg^c L + g^c g S, \tag{5}$$

$$\phi(\mathbf{15}, \mathbf{1})(\psi(\overline{\mathbf{6}}, \mathbf{2}))^2 = QH_d D^c + QH_u U^c + LH_d E^c + LH_u N^c$$
$$+ SH_u H_d + gN^c D^c + gE^c U^c + g^c U^c D^c. \tag{6}$$

From the viewpoint of the string unification theory, it is reasonable that the hierarchical structure of Yukawa couplings is attributable to some kind of the flavor symmetry at the string scale M_S. If there exists a flavor symmetry such as $U(1)_F$ in the theory, it is natural that the Froggatt-Nielsen mechanism is at work for the interactions[8]. For instance, the effective Yukawa interactions for up-type quarks are of the form[3]

$$M_{ij} Q_i U_j^c H_{u0} \tag{7}$$

with

$$M_{ij} = m_{ij} \left(\frac{\langle X \rangle}{M_S} \right)^{b_{ij}} = m_{ij} x^{b_{ij}}, \tag{8}$$

where the subscripts i and j stand for the generation indices and the coupling constants m_{ij}'s are assumed to be $\mathcal{O}(1)$ with rank $m_{ij} = 3$. $X \equiv (S_0 \overline{S})/M_S$ is singlet with a nonzero

flavor $U(1)_F$ charge and $x \equiv \langle X \rangle / M_S < 1$. The exponents b_{ij} are some non-negative integers which are settled by the flavor symmetry[*]. The hierarchical mass matrix is derived by assigning appropriate flavor charges to the matter fields.

Generally speaking, the mixing occurs not only among three generations of low energy matter fields(quarks and letons) but also beyond generations. Below the scale $\langle N_0^c \rangle$ there appear both D^c-g^c mixings and L-H_d mixings. On the other hand, U^c has no state-mixings beyond the generation mixing. This situation is of great importance to understand the characteristic features of V_{CKM} in the present model. An early attempt of explaining the CKM matrix via D^c-g^c mixings has been made in Ref.[9], in which a SUSY $SO(10)$ model was taken.

From Eq.(8) we have the up-quark mass matrix

$$
M = \begin{pmatrix} m_{11}x^{\alpha_1+\beta_1} & m_{12}x^{\alpha_1+\beta_2} & m_{13}x^{\alpha_1} \\ m_{21}x^{\alpha_2+\beta_1} & m_{22}x^{\alpha_2+\beta_2} & m_{23}x^{\alpha_2} \\ m_{31}x^{\beta_1} & m_{32}x^{\beta_2} & m_{33} \end{pmatrix}. \tag{9}
$$

The exponents α_i and β_i are determined according as the flavor $U(1)_F$ charges of matter fields and are assumed to satisfy the relations $\alpha_1 > \alpha_2 > \alpha_3 = 0$ and $\beta_1 > \beta_2 > \beta_3 = 0$. This matrix is diagonalized by the bi-unitary transformation as

$$
M^{diag} = \mathcal{V}_u^{-1} M \mathcal{U}_u. \tag{10}
$$

Using the perturbative expansion we can obtain the eigenvalues of the matrix M, which are written in light order as

$$
m_u \simeq x^{\alpha_1+\beta_1} \left| \frac{\det M_0}{\Delta(M_0)_{11}} \right|, \quad m_c \simeq x^{\alpha_2+\beta_2} \left| \frac{\Delta(M_0)_{11}}{m_{33}} \right|, \quad m_t \simeq |m_{33}|. \tag{11}
$$

Here the matrix $(M_0)_{ij}$ means m_{ij} in Eq.(9) and $\Delta(M_0)_{ij}$ is the cofactor of (ij) element of the matrix M_0. The unitary matrix $\mathcal{V}_u$ becomes

$$
\mathcal{V}_u \simeq \begin{pmatrix} 1 - \mathcal{O}(x^{2(\alpha_1-\alpha_2)}) & -x^{\alpha_1-\alpha_2} \left(\frac{\overline{m}_{21}}{\overline{m}_{11}} \right)^* & x^{\alpha_1} \frac{m_{13}}{m_{33}} \\ x^{\alpha_1-\alpha_2} \frac{\overline{m}_{21}}{\overline{m}_{11}} & 1 - \mathcal{O}(x^{2(\alpha_1-\alpha_2)}) & x^{\alpha_2} \frac{m_{23}}{m_{33}} \\ x^{\alpha_1} \frac{\overline{m}_{31}}{\overline{m}_{11}} & -x^{\alpha_2} \left(\frac{m_{23}}{m_{33}} \right)^* & 1 - \mathcal{O}(x^{2\alpha_2}) \end{pmatrix} \tag{12}
$$

with

$$
\overline{m}_{ij} \equiv (M_0^{\dagger-1})_{ij} = \left(\frac{\Delta(M_0)_{ij}}{\det M_0} \right)^*. \tag{13}
$$

---

[*]See Ref.[3] for the detail realization of the model. Here we only give the essence of the model.

Note that the 3rd column of $\mathcal{V}_u$ is proportional to the 3rd column vector $\vec{M}_3$ of M and that the 1st column of $\mathcal{V}_u$ is proportional to the 1st column vector $\vec{\overline{M}}_1$ of $(M^\dagger)^{-1}$, which is proportional to the outer product $(\vec{M}_2 \times \vec{M}_3)^{*\ \dagger}$. Another unitary matrix $\mathcal{U}_u$ is obtained by the replacement $m_{ij} \to m_{ji}^*$ and $\alpha_i \to \beta_i$ in Eq.(12) for $\mathcal{V}_u$.

We now proceed to study the down-type quark mass matrix. Due to D^c-g^c mixings the down-type quark mass matrix is expressed in terms of the 6×6 matrix

$$\widehat{M}_d = \begin{array}{c} \\ g \\ D \end{array} \begin{array}{c} g^c \qquad D^c \\ \left(\begin{array}{cc} y_S Z & y_N M \\ 0 & \rho_d M \end{array} \right) \end{array}. \tag{14}$$

Three nonzero 3×3 matrices arise from the mass terms $Z_{ij} g_i g_j^c \langle S_0 \rangle$, $M_{ij} g_i D_j^c \langle N_0^c \rangle$ and $M_{ij} Q_i D_j^c \langle H_{d0} \rangle$, where

$$Z_{ij} = (Z_0)_{ij} x^{\alpha_i + \alpha_j + \zeta} = z_{ij} x^{\alpha_i + \alpha_j + \zeta} \tag{15}$$

with $\zeta \geq 0$ and $z_{ij} = \mathcal{O}(1)$. The exponent ζ comes from the difference in the flavor $U(1)_F$ charges between the trilinear products $g_3 g_3^c S_0$ and $g_3 D_3^c N_0^c$. Here we use the notations y_S, y_N and ρ_d for the VEV's $\langle S_0 \rangle$, $\langle N_0^c \rangle$ and $\langle H_{d0} \rangle$ in units of the string scale M_S, respectively. From Eqs.(5) and (6) it is found that the matrix Z is symmetric and that the M is the same as the up-type quark mass matrix at the unification scale. Since each Yukawa coupling undergoes the radiative corrections distinctively, in the way of the renormalization group(RG) evolution to low energy region the matrices M in $\widehat{M}_d$ deviate gradually from the matrix M for up-type quark mass matrix. Since ρ_d is very small compared to y_S and y_N, the mass eigenstates for the left-handed light quarks consist almost of D-components of the quark doublet Q. On the other hand, the large mixings between D^c and g^c can occur depending on the relative magnitude of $y_S Z$ and $y_N M$. The matrix $\widehat{M}_d$ is diagonalized by the bi-unitary transformation as

$$\widehat{M}_d^{diag} = \widehat{\mathcal{V}}_d^{-1} \widehat{M}_d \widehat{\mathcal{U}}_d \tag{16}$$

The unitary matrices are

$$\widehat{\mathcal{V}}_d \simeq \left(\begin{array}{cc} \mathcal{W}_d & -\epsilon(A+B)^{-1} B \mathcal{V}_d \\ \epsilon B(A+B)^{-1} \mathcal{W}_d & \mathcal{V}_d \end{array} \right),$$

$$\widehat{\mathcal{U}}_d \simeq \left(\begin{array}{cc} y_S Z^\dagger \mathcal{W}_d \Lambda_d^{(0)-1} & -y_S^{-1} Z^{-1} \mathcal{V}_d \Lambda_d^{(2)} \\ y_N M^\dagger \mathcal{W}_d \Lambda_d^{(0)-1} & y_N^{-1} M^{-1} \mathcal{V}_d \Lambda_d^{(2)} \end{array} \right), \tag{17}$$

[†]In the previous studies([3, 4]) the same calculation for $\mathcal{V}_u$ has been carried out up to the $\mathcal{O}(1)$ factors m_{ij} and $\overline{m}_{ij}$.

where

$$\epsilon \equiv \frac{\rho_d}{y_N} \ll 1, \qquad A \equiv y_S^2 ZZ^\dagger, \qquad B \equiv y_N^2 MM^\dagger. \qquad (18)$$

Since ϵ is a very small number, the calculation is carried out by using the perturbative expansion. The unitary matrices $\mathcal{W}_d$ and $\mathcal{V}_d$ are determined such that Hermite matrices $A + B$ and $(A^{-1} + B^{-1})^{-1}$ is diagonalized by the unitary transformations as

$$(\Lambda_d^{(0)})^2 = \mathcal{W}_d^{-1}(A + B)\mathcal{W}_d, \qquad (\Lambda_d^{(2)})^2 = \mathcal{V}_d^{-1}(A^{-1} + B^{-1})^{-1}\mathcal{V}_d. \qquad (19)$$

$\Lambda_d^{(0)}$ and $\Lambda_d^{(2)}$ represent three eigenvalues for heavy modes with GUT scale masses and those for light modes corresponding to d-, s- and b-quarks, respectively. Let us assume that the (1,1) elements of A^{-1} and B^{-1} are of the same order. Since we obtain $(A^{-1})_{11} = \mathcal{O}((y_S\, x^{2\alpha_1 + \zeta})^{-2})$ and $(B^{-1})_{11} = \mathcal{O}((y_N\, x^{\alpha_1 + \beta_1})^{-2})$, this assumption implies $y_S\, x^{\alpha_1 + \zeta} \simeq y_N\, x^{\beta_1}$. Hereafter we refer this assumption to as large D^c-g^c mixings.

Under the assumption of large D^c-g^c mixings the mass eigenvalues for light quarks are given by

$$
\begin{aligned}
m_d &\simeq \frac{x^{\alpha_1 + \beta_1}}{\sqrt{|\bar{z}_{11}|^2 + |\bar{m}_{11}|^2}}, \\[2ex]
m_s &\simeq x^{\alpha_2 + \beta_1} \frac{\sqrt{|\bar{z}_{11}|^2 + |\bar{m}_{11}|^2}}{\left\| \begin{matrix} \bar{m}_{11} & \bar{z}_{11} \\ \bar{m}_{21} & \bar{z}_{21} \end{matrix} \right\|}, \\[2ex]
m_b &\simeq x^{\beta_1 - \alpha_1 + \alpha_2} \frac{|\det M_0 \cdot \det Z_0| \cdot \left\| \begin{matrix} \bar{m}_{11} & \bar{z}_{11} \\ \bar{m}_{21} & \bar{z}_{21} \end{matrix} \right\|}{\sqrt{\left\| \begin{matrix} \vec{z}_3 & \vec{m}_2 & \vec{m}_3 \end{matrix} \right\|^2 + x^{-2\eta}\left\| \begin{matrix} \vec{m}_3 & \vec{z}_2 & \vec{z}_3 \end{matrix} \right\|^2}},
\end{aligned}
\qquad (20)
$$

where $\bar{z}_{ij} \equiv (Z_0^{\dagger -1})_{ij}$, $\eta \equiv (\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)$ and $\vec{m}_i(\vec{z}_i)$ means the i-th column vector of $M_0(Z_0)$. It is noticeable that this hierarchical mass pattern of down-type quarks is rather different from that of up-type quarks. This result is in line with experimental facts. The eigenstates are approximately written as

$$
\begin{aligned}
|d\rangle &\approx \frac{1}{\sqrt{|\bar{z}_{11}|^2 + |\bar{m}_{11}|^2}}(-\bar{z}_{11}|g_1^c\rangle + \bar{m}_{11}|D_1^c\rangle), \\[2ex]
|s\rangle &\approx \frac{1}{\sqrt{|\bar{z}_{11}|^2 + |\bar{m}_{11}|^2}}(-\bar{m}_{11}^*|g_1^c\rangle - \bar{z}_{11}^*|D_1^c\rangle),
\end{aligned}
\qquad (21)
$$

$$|b\rangle \approx \frac{-|\ \vec{z}_3\ \vec{m}_2\ \vec{m}_3\ |^*|g_2^c\rangle - x^{-\eta}|\ \vec{m}_3\ \vec{z}_2\ \vec{z}_3\ |^*|D_2^c\rangle}{\sqrt{||\ \vec{z}_3\ \vec{m}_2\ \vec{m}_3\ ||^2 + x^{-2\eta}||\ \vec{m}_3\ \vec{z}_2\ \vec{z}_3\ ||^2}}.$$

The diagonalization matrix for light $SU(2)_L$-doublet down-quark sector is

$$\mathcal{V}_d \simeq \begin{pmatrix} 1 - \mathcal{O}(x^{2(\alpha_1-\alpha_2)}) & -x^{\alpha_1-\alpha_2}a_{21}^* & x^{\alpha_1}a_{13} \\ x^{\alpha_1-\alpha_2}a_{21} & 1 - \mathcal{O}(x^{2(\alpha_1-\alpha_2)}) & x^{\alpha_2}a_{23} \\ x^{\alpha_1}a_{31} & -x^{\alpha_2}a_{23}^* & 1 - \mathcal{O}(x^{2\alpha_2}) \end{pmatrix}, \tag{22}$$

where

$$a_{21} = \frac{\overline{z}_{11}^*\overline{z}_{21} + \overline{m}_{11}^*\overline{m}_{21}}{|\overline{z}_{11}|^2 + |\overline{m}_{11}|^2}, \qquad a_{13}^* = \frac{\begin{vmatrix} \overline{m}_{21} & \overline{z}_{21} \\ \overline{m}_{31} & \overline{z}_{31} \end{vmatrix}}{\begin{vmatrix} \overline{m}_{11} & \overline{z}_{11} \\ \overline{m}_{21} & \overline{z}_{21} \end{vmatrix}},$$

$$a_{31} = \frac{\overline{z}_{11}^*\overline{z}_{31} + \overline{m}_{11}^*\overline{m}_{31}}{|\overline{z}_{11}|^2 + |\overline{m}_{11}|^2}, \qquad a_{23}^* = -\frac{\begin{vmatrix} \overline{m}_{11} & \overline{z}_{11} \\ \overline{m}_{31} & \overline{z}_{31} \end{vmatrix}}{\begin{vmatrix} \overline{m}_{11} & \overline{z}_{11} \\ \overline{m}_{21} & \overline{z}_{21} \end{vmatrix}}. \tag{23}$$

It is worth noting that the 1st column of $\mathcal{V}_d$ is proportional to a linear combination of $\vec{\overline{Z}}_1$ and $\vec{\overline{M}}_1$ and that the 3rd column is to $(\vec{\overline{M}}_1 \times \vec{\overline{Z}}_1)^*$. These significant features are obtained if and only if large D^c-g^c mixings occur.

We are now in a position to calculate the mixing matrix V_{CKM}. The V_{CKM} is given by

$$V_{CKM} = \mathcal{V}_u^{-1}\mathcal{V}_d$$

$$\simeq \begin{pmatrix} 1 - \mathcal{O}(x^{2(\alpha_1-\alpha_2)}) & -x^{\alpha_1-\alpha_2}c_{21}^* & 0 \\ x^{\alpha_1-\alpha_2}c_{21} & 1 - \mathcal{O}(x^{2(\alpha_1-\alpha_2)}) & x^{\alpha_2}c_{32}^* \\ x^{\alpha_1}c_{31} & x^{\alpha_2}c_{32} & 1 - \mathcal{O}(x^{2\alpha_2}) \end{pmatrix}, \tag{24}$$

at the leading order with

$$c_{21} = \frac{\overline{z}_{11}^* \cdot \begin{vmatrix} \overline{m}_{11} & \overline{z}_{11} \\ \overline{m}_{21} & \overline{z}_{21} \end{vmatrix}}{\overline{m}_{11}(|\overline{z}_{11}|^2 + |\overline{m}_{11}|^2)},$$

$$c_{32} = \frac{\overline{m}_{11} \cdot \left| \begin{array}{ccc} \vec{m}_3 & \vec{z}_2 & \vec{z}_3 \end{array} \right|^*}{m_{33}^* \cdot (\det Z_0)^* \cdot \left| \begin{array}{cc} \overline{m}_{11} & \overline{z}_{11} \\ \overline{m}_{21} & \overline{z}_{21} \end{array} \right|}, \tag{25}$$

$$c_{31} = c_{32} \cdot c_{21}.$$

If we take the parameterization $x^{\alpha_1} = \lambda^3$, $x^{\alpha_1-\alpha_2} = \lambda$ and $c_{21}, c_{32} = \mathcal{O}(1)$, the V_{CKM} in Eq.(24) is in accord with experimental facts. It should be emphasizing that the element (1,3) of V_{CKM}, i.e., V_{ub} vanishes at the leading order. This is due to the fact that the 1st column of $\mathcal{V}_u$ is proportional to $\vec{\overline{M}}_1$ and that the 3rd column of the matrix $\mathcal{V}_d$ is proportional to $(\vec{\overline{M}}_1 \times \vec{\overline{Z}}_1)^*$. Then we must pick up the next-to-leading term to obtain a nonzero value for V_{ub}. Concretely, we obtain

$$V_{ub} \simeq x^{\alpha_1+2(\beta_1-\beta_2)} c_{13} \tag{26}$$

with

$$c_{13} = -\frac{\overline{m}_{12} \cdot \left| \begin{array}{ccc} \vec{m}_3 & \vec{z}_2 & \vec{z}_3 \end{array} \right|}{|\overline{m}_{11}|^2 \cdot (\det M_0)^* \cdot \det Z_0 \cdot \left| \begin{array}{cc} \overline{m}_{11} & \overline{z}_{11} \\ \overline{m}_{21} & \overline{z}_{21} \end{array} \right|^*}. \tag{27}$$

We obtain the order of V_{ub} to be $\sim \lambda^7$ for the parameterization $x^{\alpha_1} \sim \lambda^3, x^{\alpha_2} \sim \lambda^2, x^{\beta_1} \sim \lambda^4$ and $x^{\beta_2} \sim \lambda^2$, which reasonably reproduce the masses of u and c given in Eq.(11), with $c_{13} \sim \mathcal{O}(1)$. The magnitude of V_{ub} is too small compared to the current experimental value, which suggests $V_{ub} \sim \lambda^4$[2] as shown later. In order to obtain reasonable V_{ub} at low energies we have to take the RG effects into account. Due to the RG effects the mass matrix M for up-type quarks and those in $\widehat{M}_d$ for down-type quarks which coincide with each other at the unification scale, deviate from each other at low energies. If the RG corrections for the leading terms of Yukawa couplings are of $O(10\%)$, then the RG corrections for $\mathcal{V}_u$ and $\mathcal{V}_d$ dominate over the next-to-leading terms of them at low energies. Thus, the value of V_{ub} at low energies becomes large compared to Eq.(26) but remains small compared to V_{td} and $V_{cb} \cdot V_{us}$.

On the other hand, the (3,1) element, i.e., V_{td} has a nonzero value at the leading order. The 3rd column of $\mathcal{V}_u$ is proportional to $\vec{M}_3$. The 1st column of the matrix $\mathcal{V}_d$ is proportional to a linear combination of $\vec{\overline{Z}}_1$ and $\vec{\overline{M}}_1$ as pointed out in Eq.(23). $\vec{M}_3$ is orthogonal to $(\vec{\overline{M}}_1)^* \propto \vec{M}_2 \times \vec{M}_3$ but not to $(\vec{\overline{Z}}_1)^*$ in general. In other words, a nonvanishing leading term of V_{td} is a consequence of large D^c-g^c mixings, in which (1,1) elements of A^{-1} and B^{-1} are comparable to each other. From Eq.(24) we obtain the relations

$$V_{td} = V_{cd} \cdot V_{ts}, \tag{28}$$

$$
\begin{array}{cc}
(0.004 \sim 0.013) & (0.0076 \sim 0.0094) \\
|V_{ub}| \quad < \quad |V_{cb} \cdot V_{us}|. & \qquad (29) \\
(0.0018 \sim 0.0045) & (0.0078 \sim 0.0094)
\end{array}
$$

Current experimental values cited in the parentheses[2], strongly suggest that these relations are viable.

We studied the V_{CKM} in $SU(6) \times SU(2)_R$ model, in which D^c-g^c mixings occur in down-type quark sector but no such mixings in up-type quark sector. Nontrivial V_{CKM} is induced by large D^c-g^c mixings, which are expressed in terms of Eq.(14). In the present model V_{ub} is naturally suppressed compared to V_{td}. Introducing the flavor symmetry and Froggatt-Nielsen mechanism, we understood the reason why the V_{CKM} is nearly equal to unity but not exactly unity. Further we obtain phenomenologically viable relations (28) and (29). The conditions on $\widehat{M}_d$ for yielding the above relations are in order as

(i). The (D, D^c)-block matrix M in $\widehat{M}_d$ is exactly the same as the up-type quark mass matrix at the unification scale.

(ii). There appears null or negligibly small (D, g^c)-block matrix in $\widehat{M}_d$ relative to the (D, D^c)-block matrix.

(iii). There occur large mixings between D^c and g^c with $|\langle N_0^c \rangle| \gg |\langle H_{d0} \rangle|$.

The 1st condition is not satisfied in the $SU(5)$ GUT model because U^c and D^c belong to the different irreducible representations of $SU(5)$. Therefore, V_{ub} is not suppressed relative to V_{td} in generic $SU(5)$ GUT model. The 1st condition means that $SU(2)_R$ is contained in the unification gauge group. In $SO(10)$ GUT model the circumstances are obscure due to variations of Higgs representations.

In the next step we would like to extend this model to lepton sector to interpret the large mixing observed in neutrino oscillations[10]. In lepton sector there exist mixings between L and H_d similar to D^c-g^c mixings in quark sector. In neutral lepton sector we have to take right-handed Majorana mass matrices and the seesaw mechanism into account. This study is now in progress[11].

Acknowledgements

References

[1] See for example the recent review by H. Fritzsch and Z-Z. Xing, Prog. Part. Nucl. Phys. **45** (2000) (hep-ph/9912358) and the references cited in this article.

[2] Particle Data Group,C. Caso et. al., Eur.Phys.J. **C3**, (1998) 1-794.

[3] N. Haba, C. Hattori, M. Matsuda and T. Matsuoka, Prog. Theor. Phys. **96**, (1996) 1249.

[4] N. Haba and T. Matsuoka, Prog. Theor. Phys. **99**, (1998) 831.

[5] T. Matsuoka, Prog. Theor. Phys. **100**, (1998) 107.

[6] N. Haba, C. Hattori, M. Matsuda, T. Matsuoka and D. Mochinaga, Prog. Theor. Phys. **94**, (1995) 233.

[7] J. C. Pati and A. Salam, Phys. Rev. **D10**, (1974) 275.

[8] C. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, (1979) 277.

[9] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. **D49**, (1994) 4966.

[10] Y. Fukuda et. al., Phys. Lett. **B436**, (1998) 33; Phys. Rev. Lett. **81**, (1998) 1562; Y. Suzuki, talk given at the 17th International workshop on Weak Interactions and Neutrinos, Cape Town, South Africa, January 1999; and references their in.

[11] M. Matsuda and T. Matsuoka, in preparation.