Semileptonic Decay of B-Meson into D^{**} and the Bjorken Sum Rule

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Abstract

We study the semileptonic branching fraction of B-meson into higher resonance of charmed meson D^{**} by using the Bjorken sum rule and the heavy quark effective theory(HQET). This sum rule and the current experiment of B-meson semileptonic decay into D and D^* predict that the branching ratio into $D^{**}l\nu_l$ is about 1.7%. This predicted value is larger than the value obtained by various models.

It is well known that the heavy quark effective theory (HQET) is a very useful method to study physics of hadrons containing a heavy quark[1]. For example, the HQET is used to determine Kobayashi-Maskawa matrix element $|V_{cb}|$, ¿from experiment of semileptonic decay $B \to D^* l\nu$[2]. The attractive features of the HQET are generally verified in phenomena where both of initial and final states include ground states of heavy hadron. However, the phenomena between a ground state and an excited one, for example branching ratio of B meson semileptonic decay into the excited charmed meson state $B \to D^{**} l\nu$, where D^{**} means a higher resonance state of charmed meson, are not interpreted in various models of hadrons based on the HQET[3, 4, 5, 6, 7, 8, 9]. This discrepancy could be reduced to the models of hadrons as a composite system. It is expected that the models which interpret the B-meson branching fractions of the semileptonic decays into D, D^* and D^{**}s. At present we have no such a model. So it is meaningful to study these processes by a model-independent approach. The purpose of this short note is a test of the HQET applicability to an excited state by dealing with charmed hadron excited states D^{**} without model dependence.

In the heavy quark limit $m_Q \to \infty$, heavy quark spin and velocity are free from low energy QCD[1]. So hadron state is factorized into heavy quark $|Q\rangle$ and light degrees of freedom (*l.d.f.*) $|ldf\rangle$ as

$$|\mathrm{hadron}\rangle = |Q\rangle \otimes |ldf\rangle. \tag{1}$$

These hadrons are classified by *l.d.f.* as following

$$\begin{aligned}
(0^-, 1^-) &= |Q(\frac{1}{2}^+)\rangle \otimes |ldf(\frac{1}{2}^-)\rangle, \\
(0^+, 1^+) &= |Q(\frac{1}{2}^+)\rangle \otimes |ldf(\frac{1}{2}^+)\rangle, \\
(1^+, 2^+) &= |Q(\frac{1}{2}^+)\rangle \otimes |ldf(\frac{3}{2}^+)\rangle, \\
(1^-, 2^-) &= |Q(\frac{1}{2}^+)\rangle \otimes |ldf(\frac{3}{2}^-)\rangle,
\end{aligned} \tag{2}$$

where we use the notation as

$$(J_-^P, J_+^P) = |Q(\frac{1}{2}^+)\rangle \otimes |ldf(j^P)\rangle,$$

with

$$J_\pm = j \pm \frac{1}{2} \tag{3}$$

and $J_\pm^P$ of left hand side denotes spin-parity of heavy hadrons and j^P of right hand side is spin-parity of *l.d.f.*.

In semileptonic decay conserving light degrees of freedom like $j^P = \frac{1}{2}^- \to \frac{1}{2}^-$ such as $B(0^-)$ goes to $D(0^-)$ or $D^*(1^-)$, there are 6 independent form factors defined as

$$\begin{aligned}
\langle D(v')|J_\mu|B(v)\rangle &= f_+(y)(v+v')_\mu + f_-(y)(v-v')_\mu, \\
\langle D^*(v')|J_\mu|B(v)\rangle &= ig(y)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta - (y+1)f(y)\varepsilon_\mu^* \\
&\quad +(\varepsilon^*\cdot v)\left\{\tilde{a}_+(y)(v+v')_\mu + \tilde{a}_-(y)(v-v')_\mu\right\} \ ,
\end{aligned} \tag{4}$$

where v and v' are the velocity of B and D or D^*, respectively. As the result of heavy quark symmetry, relations among these form factors are obtained and finally there exists only one independent form factor in these processes[1]. This form factor written by $\xi(y)$ is called as Isgur-Wise function (IW function). Following the HQET we obtain only one independent form factor in each semileptonic process such as $j^P = \frac{1}{2}^- \to \frac{1}{2}^+, \frac{3}{2}^+, \frac{3}{2}^-, \ldots$[9, 10]. In this letter we denote these IW functions as ξ_E, ξ_F and ξ_G for $j^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{3}{2}^-$, respectively. The relations between form factors and IW function are given in Ref.[9].

We can derive the Bjorken sum rule on IW functions[10, 11]. In the HQET hadron state is factorized into heavy and light degrees of freedom like in Eq.(1) and the sum of transition rate between heavy hadrons H_Q and $H'_{Q'}$ is equal to transition rate of heavy quark Q into Q' as

$$\sum_{H'_{Q'}} g^{\mu\nu}\langle H'_{Q'}|J_\mu|H_Q\rangle\langle H_Q|J_\nu^\dagger|H'_Q\rangle \ = \ g^{\mu\nu}\langle Q'|J_\mu|Q\rangle\langle Q|J_\nu^\dagger|Q'\rangle \sum_{ldf'} |\langle ldf'|ldf\rangle|^2$$

$$
\begin{aligned}
&= \langle Q'|J^\mu|Q\rangle\langle Q|J^\dagger_\mu|Q'\rangle \\
&= -8y,
\end{aligned} \tag{5}
$$

where $y \equiv v\cdot v'$ and here we use the unitarity relation

$$
\sum_{ldf'} |\langle ldf'|ldf\rangle|^2 = 1. \tag{6}
$$

On the other hand in the case of heavy meson transition $B \to D_X$, we obtain

$$
\begin{aligned}
\sum_{H'=D,D^*} g^{\mu\nu}\langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\dagger_\nu|H(v')\rangle &= -4y(y+1)|\xi(y)|^2, \\
\sum_{H'=D_0^*,D_1} g^{\mu\nu}\langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\dagger_\nu|H(v')\rangle &= -4y(y-1)|\xi_E(y)|^2, \\
\sum_{H'=D_1,D_2^*} g^{\mu\nu}\langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\dagger_\nu|H(v')\rangle &= -\frac{8}{3}y^2(y^2-1)(y+1)|\xi_F(y)|^2, \\
\sum_{H'=D_1^*,D_2} g^{\mu\nu}\langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\dagger_\nu|H(v')\rangle &= -\frac{8}{3}y^2(y^2-1)(y-1)|\xi_G(y)|^2, \\
\sum_{H'=D_{C_2},D^*_{C_2}} g^{\mu\nu}\langle H'(v')|J_\mu|B(v)\rangle\langle B(v)|J^\dagger_\nu|H(v')\rangle &= -4y(y+1)|\xi_{C_2}(y)|^2,
\end{aligned} \tag{7}
$$

by the straight calculation[12], where index C_2 denotes radial excitation of $j^P = \frac{1}{2}^-$. Here, we introduce the hypothesis of resonance saturation which means that it is possible to neglect the contribution from the continuum spectra to semileptonic decay. Under this assumption the relation

$$
\begin{aligned}
1 &= \frac{y+1}{2}\left(|\xi(y)|^2 + |\xi_{C_2}(y)|^2\right) + \frac{y-1}{2}|\xi_E(y)|^2 + \frac{1}{3}y(y^2-1)(y+1)|\xi_F(y)|^2 \\
&\quad + \frac{1}{3}y(y^2-1)(y-1)|\xi_G(y)|^2 + \cdots
\end{aligned} \tag{8}
$$

is derived by the comparison of Eqs.(5) and (7), where dots means contributions from other higher resonances. This is the simplified Bjorken sum rule[10, 11].

In B meson semileptonic decay, differential decay rate is given as,

$$
\frac{d\Gamma_X}{dy} \equiv \frac{d\Gamma}{dy}(B\to D_X l\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3} m_B^2\sqrt{y^2-1}\, m_{D_X}^3 W_X(y,r_X)|\xi_X(y)|^2\,, \tag{9}
$$

where lepton mass is neglected and $W_X(y, r_X)$ is a calculable functions of y and $r_X = \frac{m_{D_X}}{m_B}$[9]. In the following analysis, We assume that the excited states, which contribute to the simplified Bjorken sum rule, saturate the B-meson semileptonic decay rate(resonance saturation hypothesis) and that these exited states occur in a mass range being small compared with m_c. The latter assumption leads us to replace $m_{D_X}(X = C_2, E, F, G, \ldots)$ by a common mass $m_{D^{**}}$. Then we can sum up these decay rates for all D^{**} and we get the equality

$$\begin{aligned}
\frac{d\Gamma_{**}}{dy} &\equiv \frac{d\Gamma}{dy}(B \to D^{**} l\nu) \\
&= \sum_{X=C_2,E,F,G,\ldots} \frac{d\Gamma_X}{dy} \\
&= \frac{G_F^2 |V_{cb}|^2}{48\pi^3} m_B^2 \sqrt{y^2-1} \sum_{X=C_2,E,F,G,\ldots} m_{D_X}^3 W_X(y, r_X) |\xi_X(y)|^2 \\
&= \frac{G_F^2 |V_{cb}|^2}{24\pi^3} m_B^2 \sqrt{y^2-1} m_{D^{**}}^3 \\
&\times \left[(y-1)(1+r)^2 + (y+1)(1-r)^2 + 4y(1+r^2-2ry)\right] \\
&\times \left[\frac{y+1}{2}|\xi_{C_2}(y)|^2 + \frac{y-1}{2}|\xi_E(y)|^2 + \frac{1}{3}y(y^2-1)(y+1)|\xi_F(y)|^2 \right. \\
&\quad \left. + \frac{1}{3}y(y^2-1)(y-1)|\xi_G(y)|^2 + \cdots\right]. \qquad (10)
\end{aligned}$$

By using Eq.(8), we obtain the D^{**} contribution as

$$\begin{aligned}
\frac{d\Gamma_{**}}{dy} &= \frac{G_F^2 |V_{cb}|^2}{24\pi^3} m_B^2 \sqrt{y^2-1} m_{D^{**}}^3 \\
&\quad \times \left[(y-1)(1+r)^2 + (y+1)(1-r)^2 + 4y(1+r^2-2ry)\right] \\
&\quad \times \left[1 - \frac{y+1}{2}|\xi(y)|^2\right]. \qquad (11)
\end{aligned}$$

¿From this result we can estimate the D^{**} contribution by using the following parameters[13]

$$V_{cb} = 0.040, \qquad m_B = 5.279\,\mathrm{GeV}, \qquad \tau_B = 1.537\,ps \qquad (12)$$

and for the mass of D^{**} we use the following weighted average mass

$$m_{D^{**}} = \frac{3m_{D_1} + 5m_{D_2^*}}{8} = 2.444\,\mathrm{GeV} \qquad (13)$$

with $m_{D_1} = 2.420$GeV and $m_{D_2^*} = 2.460$GeV. We use the IW function $\xi(y)$ with following three trial functions

$$\begin{array}{lll} \text{(I)} & 1-\rho^2(y-1) & \rho = 0.91^{+0.19}_{-0.21}, \\ \text{(II)} & \exp[-\rho^2(y-1)] & \rho = 0.99^{+0.27}_{-0.25}, \\ \text{(III)} & \left(\frac{2}{y+1}\right)^{2\rho^2} & \rho = 1.03^{+0.28}_{-0.26}, \end{array}$$

where ρ is a free parameter to reproduce the experimental branching ratio

$$\begin{aligned} \mathrm{Br}(B \to D^{(*)} l\nu) &= \mathrm{Br}(B \to D l\nu) + \mathrm{Br}(B \to D^* l\nu) \\ &= (1.6 \pm 0.7)\,\% + (6.6 \pm 2.2)\,\% \\ &= (8.2 \pm 2.3)\,\%, \end{aligned}$$

and we use $m_{D^{(*)}} = \frac{m_D + 3m_{D^*}}{4} = 1.975$GeV to determine ρ.

Table 1: Branching ratio $B \to D_X l\nu$ $\left(|\frac{V_{cb}}{0.040}|^2 \frac{\tau_B}{1.537\,ps}\,\%\right)$

	D	D^*	D^{**}	$\sum_X D_X$
(I)	$1.91\,^{-0.68}_{+0.76}$	$6.24\,^{-1.53}_{+1.63}$	$1.58\,^{+0.89}_{-0.86}$	$9.73\,^{-1.33}_{+1.52}$
(II)	$1.95\,^{-0.67}_{+0.72}$	$6.22\,^{-1.61}_{+1.61}$	$1.71\,^{+1.05}_{-0.93}$	$9.88\,^{-1.24}_{+1.40}$
(III)	$1.96\,^{-0.67}_{+0.72}$	$6.21\,^{-1.62}_{+1.62}$	$1.77\,^{+1.06}_{-0.95}$	$9.94\,^{-1.23}_{+1.39}$
Average	$1.94\,^{-0.68}_{+0.73}$	$6.22\,^{-1.59}_{+1.62}$	$1.69\,^{+1.00}_{-0.91}$	$9.85\,^{-1.27}_{+1.44}$

The D^{**} contribution and D_X total contribution $(D + D^* + D^{**})$ are given in Table 1. Here we use Eq.(11) to obtain D^{**} contribution which gives about 1.7% branching fraction. The direct measurement of D^{**} contribution is $2.7 \pm 0.7\%$[13]. This is a factor 1.6 larger at central value than the theoretical estimation of D^{**} obtained by the simplified Bjorken sum rule Eq.(8). However, the experimental error is still large and the estimated magnitude of D^{**} contribution seems to be consistent with the present experiment. On the other hand, inclusive semileptonic decay branching ratio is experimentally $10.43 \pm 0.24\%$ and the unidentified semileptonic

Table 2: Comparison of D^{**} contribution $\left(|\frac{V_{cb}}{0.040}|^2 \frac{\tau_B}{1.537\,ps}\,\%\right)$

	ISGW2[7]	SISM[9]	CNP[4]	VO[8]	ours*	Exp.[13]
$\mathrm{Br}(D+D^*)$	9.03	7.23	7.24	9.24	$8.16^\dagger$	8.2± 2.3
$\mathrm{Br}(D^{**})$	0.96	0.33	0.53	0.96	1.69	2.7± 0.7

*In this table ours data are average in Table 1.

†This is an input value.

branching fraction [13] is

$$\mathrm{Br}(B \to \mathrm{unknown}) = \mathrm{Br}(\mathrm{inclusive}) - \mathrm{Br}(B \to D \mathrm{and} D^* l\nu) = 2.2 \pm 2.3\,\%. \quad (14)$$

This also shows that the resonance saturation hypothesis and the approximate mass degeneracy among the excited charmed mesons($D_0^*, D_1, D_1^*, D_2, D_2^*, D_{c_2}, D_{c_2}^* \cdots$) might hold in B meson semileptonic decay.

In order to estimate the contribution from each resonance, it is necessary to calculate exclusive decay processes by using hadronic models as given in Ref.[3, 4, 5, 6, 7, 8, 9]. Through these studies there is a tendency that branching fractions into D_2^* and D_1 are rather larger compared to the other excited states. The maximum fraction among D^{**}s is D_2^* in Ref.[4, 8, 9] with the magnitude $0.1\% \sim 0.4\%$ and is D_1 in Ref.[7] with 0.4%. To make clear which model is better, further exclusive experiments are needed. One more feature in common with model-dependent analyses given in Table 2 is that relatively small D^{**} fraction is predicted and is inconsistent with experimental value.

Further in order to check our model-independent approach we apply this method to B_s semileptonic decay processes. The IW function of $B_s \to D_s^{(*)} l\nu$ is the same one of $B \to D^{(*)} l\nu$, because u, d and s quarks are treated as *l.d.f.* in the HQET. In the numerical estimation the parameters are[13]

$$\begin{aligned} m_{B_s} &= 5.375\,\mathrm{GeV}, \qquad \tau_{B_s} = 1.34\,ps, \\ m_{D_s^{(*)}} &= \frac{m_{D_s} + 3m_{D_s^*}}{4} = 2.075\,\mathrm{GeV}, \end{aligned}$$

$$m_{D_s^{(**)}} \;=\; \frac{3m_{D_{s1}} + 5m_{D_{s2}^*}}{8} = 2.556\,\mathrm{GeV}\,,$$

where $m_{D_{s2}^*} \cong 2568$MeV is a presumption from other charmed meson masses by using the relation $m_{D_{s2}^*} - m_{D_{s1}}(2535) = m_{D_2^*}(2456) - m_{D_1}(2423)$. Following the same argument of $B \to D_X l\nu$ we can give the branching ratios of B_s to D_s, D_s^* and D_s^{**} in Table 3. The predicted branching ratio is similar to ones of $B \to D_X l\nu$ as shown in Table 1. The largest branching ratio is reduced to a fraction to D_s^* and a contribution from excited states D_s^{**} is less than 16% of inclusive ratio. If this is confirmed by experiments it will verify the validity of the analysis using the Bjorken sum rule.

Table 3: Branching ratio $B_s \to D_{sX} l\nu$ $\left(|\frac{V_{cb}}{0.040}|^2 \frac{\tau_{B_s}}{1.34\,ps}\,\%\right)$

	D_s	D_s^*	D_s^{**}	$\sum_X D_{sX}$
(I)	$1.79\ {}^{-0.59}_{+0.64}$	$5.78\ {}^{-1.31}_{+1.37}$	$1.28\ {}^{+0.73}_{-0.70}$	$8.84\ {}^{-1.17}_{+1.31}$
(II)	$1.80\ {}^{-0.59}_{+0.61}$	$5.72\ {}^{-1.41}_{+1.38}$	$1.40\ {}^{+0.88}_{-0.76}$	$8.92\ {}^{-1.12}_{+1.23}$
(III)	$1.81\ {}^{-0.59}_{+0.61}$	$5.70\ {}^{-1.42}_{+1.39}$	$1.45\ {}^{+0.89}_{-0.78}$	$8.96\ {}^{-1.12}_{+1.22}$
Average	$1.80\ {}^{-0.59}_{+0.62}$	$5.73\ {}^{-1.38}_{+1.38}$	$1.38\ {}^{+0.83}_{-0.75}$	$8.91\ {}^{-1.14}_{+1.25}$

In this letter we estimate the semileptonic branching ratios of B to excited charmed mesons by using a method independent on specific hadron models. The Bjorken sum rule will be checked by measuring the contribution ¿from higher resonance of charmed meson in semileptonic decay of $B_{u,d}$ and B_s meson. The prediction is given under the assumption that the semileptonic decay is saturated by the three body decays $B \to D_X l\nu$ and the continuum contribution is negligible and that the excited states have approximately equal masses. We get Br$(B \to D^{**} l\nu)$ = 1.7± 1.0% which seems to be consistent with the experimental value 2.7± 0.7%. We also estimate Br$(B_s \to D_s, D_s^*, D_s^{**} l\nu)$ to be 1.8%, 5.7% and 1.4%, respectively. The predicted branching ratios are similar to Br$(B \to D, D^*, D^{**} l\nu)$. These evaluations are to be checked by experiments in near future and we expect that the model-independent approach will be confirmed experimentally.

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