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# Zee Neutrino Mass Matrix in the Gauge Mediated Supersymmetry Breaking Scenario

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**Abstract**

It is well known that Zee type neutrino mass matrix can provide bi-maximal neutrino mixing for three neutrinos. We study the reconciliation of this model with the gauge mediated supersymmetry breaking scenario, which naturally suppresses the large flavor changing neutral current and CP violation in the supersymmetric standard model. When the messenger fields have suitable $B-L$ charges, the radiative correction naturally induces the Zee neutrino mass matrix, which provides tiny neutrino masses and large lepton flavor mixings. Our numerical results are consistent with the neutrino oscillation experiments in both three and four neutrino models.

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Recent neutrino oscillation experiments provide a strong evidence of tiny neutrino masses and large lepton flavor mixings[1, 2, 3]. We know two mechanisms which can explain in a natural way the smallness of neutrino masses. One is the see-saw mechanism which can induce the small neutrino mass by integrating out heavy right-handed neutrinos[4]. The second scenario is that neutrinos obtain their masses by the radiative corrections through which the left-handed neutrinos obtain the Majorana masses. The latter yields small neutrino masses radiatively. A typical example is the so-called Zee model[5], which does not need right-handed neutrinos[6]. The original Zee model is not embedded into GUT or supersymmetry (SUSY). Some authors have tried to embed it into SUSY with $R$-parity breaking model[7] since the right-handed slepton in SUSY has favorable quantum number to play the role of Zee-singlet which is charged singlet scalar under the standard model(SM). In these scenarios the neutrino masses strongly depend on $R$-parity violating parameters in the SUSY Lagrangian and the Zee mass matrix is derived by the artificial adjustment of the parameters.

In this paper, we present an attractive way to embed the Zee model into $R$-parity conserving SUSY model. It is well known that the gauge mediated SUSY breaking mechanism is one of the most reliable scenarios. The messenger field of SUSY-breaking can play the role of Zee-singlet which leads to Zee neutrino mass matrix. Then the neutrino masses are given in terms of the SUSY breaking parameters.

If there is no right-handed neutrino, neutrinos are unable to obtain the Dirac mass terms as in the SM. In order to obtain neutrino masses without right-handed neutrinos in the SUSY model the following three conditions are required:
$(i)$ : $SU(2)_L$ must be broken,
$(ii)$ : lepton number must be broken,
$(iii)$ : supersymmetry must be broken.
Quarks and leptons can not obtain their masses without the first condition. The second condition is required to obtain Majorana neutrino masses. The lepton number conservation prevents neutrinos from obtaining the Majorana masses in the SM. Recall that neutrinos can obtain masses in the $R$-parity breaking scenario in the SUSY theory[10]. This is due to the fact that $R$-parity is broken whenever the lepton number is broken. The conditions $(i)$ and $(ii)$ are also needed in the see-saw mechanism which includes the right-handed neutrinos. The SUSY non-renormalization theorem requires the third condition $(iii)$ since the neutrino masses are generated by the $F$-terms in the SUSY theory. If the SUSY is the exact symmetry, neutrinos can not obtain masses from the quantum corrections.

Let us consider the low energy gauge mediated SUSY breaking mechanism. We can show that radiative corrections induce the tiny neutrino masses and large lepton flavor mixing if the messenger fields have suitable $B-L$ charges, extra Higgs doublets, and two singlet fields have lepton number. If three extra singlet fields and one more pair of the messenger fields are added then four neutrino scenario is also realized which in particular explains LSND experiment[11].

Taking into account the gauge mediated SUSY breaking scenario, we investigate the possibility to obtain the small neutrino masses by quantum corrections. Gauge interaction plays the role of the messenger of SUSY breaking in a gauge mediated SUSY breaking scenario, in which the flavor changing neutral current(FCNC) and also $CP$ violation through the couplings of SUSY particles are naturally suppressed. We introduce the singlet field $\phi$ under $SU(5)$, which is required to generate soft mass terms. This field has $F$-term as

$$\phi = \langle\phi\rangle + \langle F_\phi\rangle\,\theta^2, \tag{1}$$

and SUSY breaking effects are mediated to the low energy by the couplings with messenger fields. The messenger fields $\mathbf{10_M} + \overline{\mathbf{10_M}}$ are introduced in the $SU(5)$ representation with the ordinary quantum charge for the SM gauge symmetry, which component are given by

$$\mathbf{10_M} = (Q_M, \overline{U_M}, \overline{E_M}), \qquad \overline{\mathbf{10_M}} = (\overline{Q_M}, U_M, E_M). \tag{2}$$

They can mediate universal soft SUSY breaking parameters through $\phi$ by the flavor blind gauge interactions[8] *. The squark and slepton soft masses, and gaugino masses are given by $(\alpha/4\pi)(\langle F_\phi\rangle/\langle\phi\rangle) = \mathcal{O}(10^2)$GeV, where $\alpha$ denotes gauge coupling. On the other hand, scalar three point soft breaking terms ($A$-terms) are induced by the two-loop diagrams and their magnitudes are estimated as $(\alpha/4\pi)^2(\langle F_\phi\rangle/\langle\phi\rangle) = \mathcal{O}(1)$GeV.

The matter fields are given by

$$\mathbf{10_f} = (Q, \overline{U}, \overline{E}), \qquad \overline{\mathbf{5_f}} = (\overline{D}, L), \tag{3}$$

which are the same as the conventional $SU(5)$ grand unified gauge theory. The Higgs fields are given by

$$\Phi = (C, H), \qquad \overline{\Phi} = (\overline{C}, \overline{H}), \tag{4}$$

$$\Phi_e = (C_e, H_e), \qquad \overline{\Phi_e} = (\overline{C_e}, \overline{H_e}), \tag{5}$$

* The universality of soft SUSY breaking parameters is modified, when there are messenger-matter mixings. This is because the Yukawa interactions can also mediate SUSY breaking parameters[9]. Actually, soft scalar masses in our following models are shifted by the effects of messenger-matter mixings as discussed later.

where triplets are colored Higgs $C, \overline{C}, C_e, \overline{C_e}$. These fields must be heavy enough to avoid rapid proton decay. The $H$ and $\overline{H}$ are the ordinary Higgs particles, and $H_e$ and $\overline{H_e}$ are the extra Higgs doublets. We also introduce two gauge singlet fields $\chi$ and $\overline{\chi}$ with non-zero lepton number. These fields are required in this scenario to obtain reasonable vacuum expectation values of $\mathcal{O}(10^2)$ GeV for $H$, $\overline{H}$, $H_e$, and $\overline{H_e}$. The extra fields $(\mathbf{10_M} + \overline{\mathbf{10_M}})$ and $(\Phi_e + \overline{\Phi_e})$ have the conventional gauge quantum numbers except for unusual $B-L$ charges $Q_{B-L}$ as

$$Q_{B-L} = Q_F + \frac{2}{5}Y, \tag{6}$$

where $Y$ is the ordinary hypercharge, and $Q_F$-charge for the relevant fields is given in Table 1.

| Field | $\Phi$ | $\overline{\Phi}$ | $\overline{\mathbf{5_f}}$ | $\mathbf{10_f}$ | $\mathbf{10_M}$ | $\overline{\mathbf{10_M}}$ | $\Phi_e$ | $\overline{\Phi_e}$ | $\chi$ | $\overline{\chi}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $Q_F$ | $-\frac{2}{5}$ | $\frac{2}{5}$ | $-\frac{3}{5}$ | $\frac{1}{5}$ | $\frac{6}{5}$ | $-\frac{6}{5}$ | $\frac{8}{5}$ | $-\frac{8}{5}$ | $2$ | $-2$ |
| $Z_2$ | $+$ | $+$ | $-$ | $-$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |

Table 1: $Q_F$-charge in three neutrino model

In Table 1 we also introduce $Z_2$ symmetry which distinguishes matter fields with Higgs and messenger fields $^\dagger$. From the charge assignments in Table 1, we obtain the superpotential $W_3$ in three neutrino model as

$$\begin{aligned} W_3 &= \mathbf{10_f}\mathbf{10_f}\Phi + \mathbf{10_f}\overline{\mathbf{5_f}}\,\overline{\Phi} + \mathbf{10_M}\overline{\mathbf{5_f}}\,\overline{\mathbf{5_f}} + \mathbf{10_M}\overline{\Phi}\,\overline{\Phi_e} + \overline{\mathbf{10_M}}\Phi\Phi_e \\ &\quad + M\mathbf{10_M}\overline{\mathbf{10_M}} + \chi\Phi\overline{\Phi_e} + \overline{\chi}\Phi_e\overline{\Phi} + \mu\Phi\overline{\Phi} + \mu_e\Phi_e\overline{\Phi_e} + \mu_\chi\chi\overline{\chi}, \end{aligned} \tag{7}$$

where the 1st and 2nd terms are usual Yukawa interactions, and the 3rd and 4th terms denote the couplings of messenger field with matter and Higgs fields, respectively. These couplings are the origin to yield Zee neutrino mass matrix radiatively. The 5th term is conjugate to the 4th term. The 6th term corresponds to the mass term of messenger field, and $M$ is the order of the messenger scale induced by $\langle\phi\rangle$. The remaining terms in $W_3$ generate Higgs scalar potential with taking weak scale order for $\mu$'s. The superpotential $W_3$ preserves $U(1)_{B-L}$ global symmetry and $Z_2$ discrete symmetry.

$^\dagger$ $Z_2$ symmetry is the extension of the conventional $R$-parity, which distinguishes not only the matter field $\overline{\mathbf{5_f}}$ $(L)$ and the Higgs field $\overline{\Phi}$ $(\overline{H})$, but also the matter field $\mathbf{10_f}$ and the messenger field $\mathbf{10_M}$. Without $Z_2$ symmetry this model is similar to the $R$-parity breaking scenario, where neutrinos can obtain their masses through the mixing with neutralinos.

Notice the importance of the proton decay. Due to $U(1)_{B-L}$ symmetry, the dominant operator to cause proton decay is the dimension five operator $(QQQL)$ or $(\overline{D}\,\overline{D}\,\overline{D}\,\overline{E})$ mediated by the colored Higgs $C$'s or $\overline{C}$'s exchange. We assume the triplet-doublet splittings in the Higgs sector, where colored Higgs fields have super-heavy masses enough to avoid the rapid proton decay, while Higgs doublets $H$'s and $\overline{H}$'s have weak scale masses. Thus the proton decay is suppressed enough in this model as in the ordinary grand unified models. The gauge unification of SUSY seems to be destroyed by the existence of two extra light Higgs doublets $H_e$ and $\overline{H_e}$. However, the introduction of missing partner fields could recover the gauge coupling unification and also resolve the triplet-doublet splitting[12].

Take non-zero vacuum expectation values $\langle H_e \rangle$, $\langle \overline{H_e} \rangle$, $\langle \chi \rangle$, and $\langle \overline{\chi} \rangle$ at the weak scale in this model. Then $U(1)_{B-L}$ symmetry is spontaneously broken and massless Majoron particles should appear. However, Majoron fields can almost decouple not only with quarks and leptons but also with gauge bosons, since Majoron fields can be almost composed of singlet fields in this model. Therefore the existence of massless Majoron in this model might be compatible with accelerator experiments.

Now let us estimate the neutrino masses and lepton flavor mixing angles according to Eq.(7). We extract the interaction of the lepton doublet in Yukawa coupling in the 3rd term. This term is denoted by $f_{\alpha\beta}\,\overline{E_M} L_\alpha L_\beta$, where indices correspond to the flavor: $\alpha, \beta = e, \mu, \tau$. Since the coupling $f_{\alpha\beta}$ is antisymmetric by statics, the diagonal elements of the neutrino mass matrix $M_\nu$ become zero. The neutrino mass matrix is generated by Fig.1 radiatively at one loop level and we obtain

$$M_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}. \tag{8}$$

This is Zee type of neutrino mass matrix which leads to the stable lepton flavor mixing matrix[13] against the quantum corrections[14]. The neutrino masses are given as

$$\begin{aligned} m_{e\mu} &= f_{e\mu}(m_\mu^2 - m_e^2) A \frac{\langle \overline{H_e} \rangle}{\langle \overline{H} \rangle} F(M^2, \mu^2), \\ m_{e\tau} &= f_{e\tau}(m_\tau^2 - m_e^2) A \frac{\langle \overline{H_e} \rangle}{\langle \overline{H} \rangle} F(M^2, \mu^2), \\ m_{\mu\tau} &= f_{\mu\tau}(m_\tau^2 - m_\mu^2) A \frac{\langle \overline{H_e} \rangle}{\langle \overline{H} \rangle} F(M^2, \mu^2), \end{aligned} \tag{9}$$

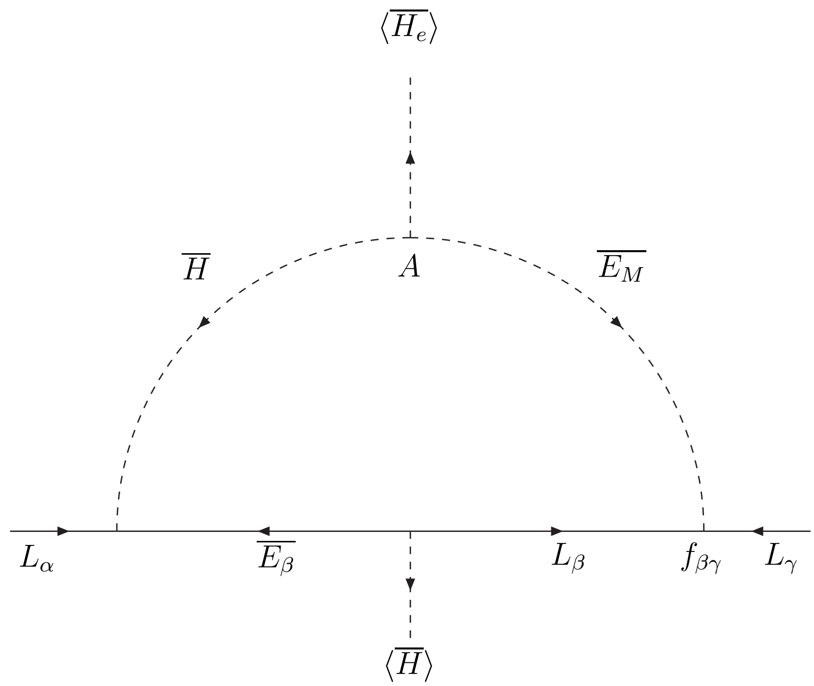


Figure 1: Feynman diagram to generate neutrino masses in the three neutrino model

where

$$F(M^2, \mu^2) = \frac{1}{16\pi^2} \frac{1}{M^2 - \mu^2} \ln \frac{M^2}{\mu^2}. \tag{10}$$

Here $A$ is the soft mass of the scalar three point coupling $\overline{E_M}\ \overline{H}\ \overline{H_e}$ with $\mathcal{O}(1)$GeV. The unique mass matrix compatible with the solar and the atmospheric experiments in the Zee model with three neutrino, requires (1,2) and (1,3) elements to be of the same order, and (2,3) element to be negligible when compared to (1,2) and (1,3) elements[15]. When $f_{e\mu} \gg f_{e\tau} \gg f_{\mu\tau}$ and $f_{e\mu}/f_{e\tau} \simeq m_\tau^2/m_\mu^2$, the neutrino mass matrix $M_\nu$ in Eq.(8) can induce the bi-maximal mixings, suggesting the atmospheric neutrino solution and solar vacuum solution or large angle MSW solution. The bi-maximal condition $0.02\text{eV} < m_{e\mu} < 0.08\text{eV}$ is realized[15], when $f_{e\mu} \sim 1$, $M \sim 10^{4.5}$ GeV ‡ and $\langle \overline{H} \rangle < \langle \overline{H_e} \rangle$.

‡ The messenger scale of $M$ and $M_0$ in Eq.(12) must be larger than $10^4$ GeV. This lower bound is required by the positivity of the scalar masses of the messenger quarks and leptons. We thank Y. Mimura and Y. Nomura for this observation.

In order to reconcile the data of LSND experiment as well as solar and atmospheric experiments, we need three flavor and one sterile neutrinos at least. It is meaningful to construct the messenger model with four tiny neutrino masses by quantum corrections. Let us introduce another $(\mathbf{10^0_M} + \overline{\mathbf{10^0_M}})$ messenger fields [§] in the $SU(5)$ gauge representation, which are denoted by

$$\mathbf{10^0_M} = (Q^0_M, \overline{U^0_M}, \overline{E^0_M}), \qquad \overline{\mathbf{10^0_M}} = (\overline{Q^0_M}, U^0_M, E^0_M). \tag{11}$$

Since these fields have ordinary quantum charges for the SM gauge symmetry, they can mediate SUSY breaking through $\phi$ in Eq.(1) by the conventional gauge mediated scenario. We also introduce three gauge singlet fields $S$, $N$, and $\overline{N}$ which have the lepton number. $S$ corresponds to the sterile neutrino. The extra fields $(\mathbf{10_M} + \overline{\mathbf{10_M}})$, $(\mathbf{10^0_M} + \overline{\mathbf{10^0_M}})$, $S$, $N$, and $\overline{N}$ in the four neutrino case have $Q_F$ charges as listed in Table 2 together with the relevant fields in the three neutrino case. Here we also introduce $Z_3$ symmetry, which

| Field | $\Phi$ | $\overline{\Phi}$ | $\overline{\mathbf{5_f}}$ | $\mathbf{10_f}$ | $\mathbf{10_M}$ | $\overline{\mathbf{10_M}}$ | $\Phi_e$ | $\overline{\Phi_e}$ | $\chi$ | $\overline{\chi}$ | $S$ | $N$ | $\overline{N}$ | $\mathbf{10^0_M}$ | $\overline{\mathbf{10^0_M}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Q_F$ | $-\frac{2}{5}$ | $\frac{2}{5}$ | $-\frac{3}{5}$ | $\frac{1}{5}$ | $\frac{6}{5}$ | $-\frac{6}{5}$ | $\frac{8}{5}$ | $-\frac{8}{5}$ | $2$ | $-2$ | $-1$ | $-2$ | $2$ | $-\frac{4}{5}$ | $\frac{4}{5}$ |
| $Z_2$ | $+$ | $+$ | $-$ | $-$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $-$ | $+$ | $+$ | $+$ | $+$ |
| $Z_3$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | $\omega$ | $\omega$ | $\omega^2$ | $\omega$ | $\omega^2$ |

Table 2: $Q_F$-charge in four neutrino model

avoids the tree level mass of sterile neutrino $S$, for example, through the term $\chi\chi S$. The charge assignment in Table 2 determines the superpotential $W_4$ in the four neutrino model as

$$W_4 \;=\; W_3 + S\mathbf{10_f}\overline{\mathbf{10^0_M}} + N\mathbf{10_M}\overline{\mathbf{10^0_M}} + M_0\mathbf{10^0_M}\overline{\mathbf{10^0_M}} + \mu_N N\overline{N}, \tag{12}$$

where $W_3$ is given by Eq.(7). The 2nd and 3rd terms give the mass mixings between sterile and active neutrinos as shown in Fig.2. The 4th term is the mass term of $\mathbf{10^0_M}$ and $\overline{\mathbf{10^0_M}}$ and $M_0$ is of the same order as the messenger scale. The last term is also the mass term of $N$ and $\overline{N}$, where $\mu_N$ is settled around the weak scale. The superpotential $W_4$ preserves $Z_2 \times Z_3$ symmetry. In the four neutrino model we also assume that the triplet-doublet splittings are realized in the Higgs sector, and the Higgs fields $H, \overline{H}, H_e, \overline{H_e}$ and the singlet field $N$ have vacuum expectation values of order of the weak scale.

[§] In this case, the gauge couplings blow up around $10^{14}$ GeV[16].

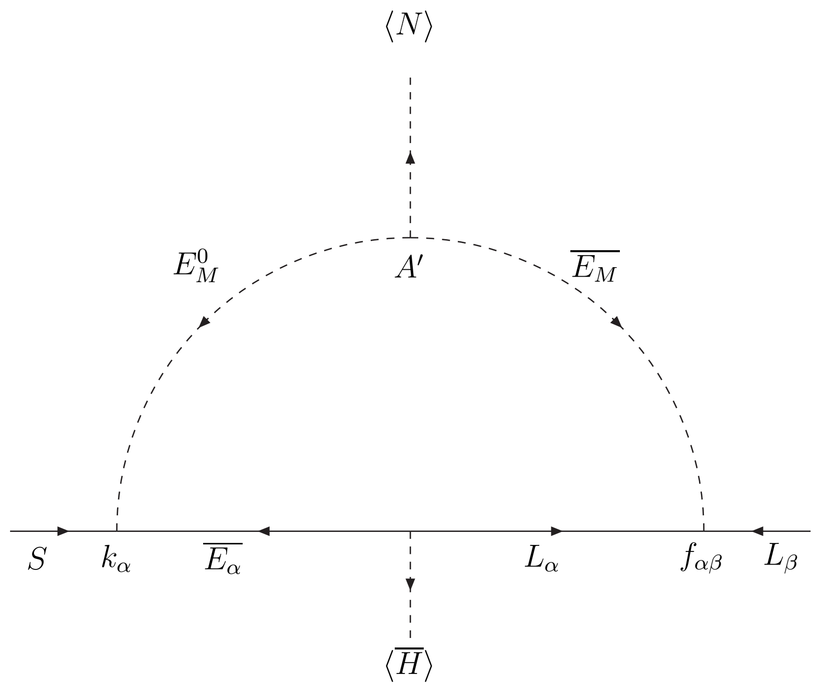


Figure 2: Feynman diagram to generate neutrino masses in the four neutrino model

Next we estimate the neutrino masses and lepton flavor mixing angles in the four neutrino model. We extract the interaction of the lepton due to the 2nd term in Eq.(12), which is denoted as $k_\alpha S\overline{E_\alpha}E_M^0$. The $4\times 4$ neutrino mass matrix is given by

$$M_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} & m_{es} \\ m_{e\mu} & 0 & m_{\mu\tau} & m_{\mu s} \\ m_{e\tau} & m_{\mu\tau} & 0 & m_{\tau s} \\ m_{es} & m_{\mu s} & m_{\tau s} & 0 \end{pmatrix}. \tag{13}$$

The mass terms including the sterile neutrino field $S$[17] are obtained from the diagram of Fig.2 as

$$\begin{aligned} m_{es} &= (f_{e\tau}k_\tau m_\tau + f_{e\mu}k_\mu m_\mu)A'\langle N\rangle F(M^2, M_0^2), \\ m_{\mu s} &= (f_{\mu\tau}k_\tau m_\tau + f_{\mu e}k_e m_e)A'\langle N\rangle F(M^2, M_0^2), \\ m_{\tau s} &= (f_{\tau\mu}k_\mu m_\mu + f_{\tau e}k_e m_e)A'\langle N\rangle F(M^2, M_0^2), \end{aligned} \tag{14}$$

where $A'$ is the soft mass of the scalar three point coupling $N\overline{E_M}E^0_M$, which is of order 1 GeV. The same type of mass matrix as in Eq.(13) was already analyzed in details in Ref.[18].

When $f_{e\mu} \ll f_{e\tau} < f_{\mu\tau}$ and $k_\tau \ll k_\mu \leq k_e$, this model can explain the LSND as well as atmospheric and the solar neutrino experiments, where the small mixing angle MSW solution is preferred. For example, when $f_{\mu\tau} \simeq 1$, $f_{e\tau} \sim 0.1$, $f_{e\mu} \simeq 10^{-3}$, $k_e = k_\mu \simeq 1$ and $k_\tau \simeq 10^{-3}$ the elements might be $m_{\mu\tau} = 0.5$ eV, $m_{e\tau} = 0.05$ eV, $m_{e\mu} = 10^{-5}$ eV, $m_{\tau s} = 0.15$ eV, $m_{\mu s} = 0.0036$ eV, and $m_{es} = 0.00025$ eV, which in turn, induce suitable neutrino mass squared differences $\delta m^2_{\rm sol} = 4 \times 10^{-6}$ eV$^2$, $\delta m^2_{\rm atm} = 2 \times 10^{-3}$ eV$^2$, $\delta m^2_{\rm LSND} = 0.3$ eV$^2$, and mixing angles $\sin^2 2\theta_{\rm sol} = 1 \times 10^{-3}$, $\sin^2 2\theta_{\rm atm} = 0.9$, $\sin^2 2\theta_{\rm LSND} = 0.03$[18].

Let us summarize the results. The Zee neutrino mass matrix is naturally realized in the three neutrino scenario in the frame of SUSY theory provided that the messenger fields have suitable $B - L$ charges, and extra Higgs doublets ($H_e$ and $\overline{H_e}$) and two singlet fields ($\chi$ and $\overline{\chi}$) have lepton numbers. If three extra singlet fields and one more pair of the messenger fields are added then the four neutrino scenario is realized. This mass matrix is consistent with solar, atmospheric and LSND experiments when the parameters are chosen appropriately.

We shall comment now on the messenger-matter mixings. The most advantage of gauge mediated SUSY breaking is the natural derivation of universal soft masses which are flavor blind as well known[9]. However, the interaction $f_{\alpha\beta}10_M\overline{5}_{f_\alpha}\overline{5}_{f_\beta}$ induces the mixing between messenger and matter field and the universality of soft mass might be broken by this term. Especially the soft mass terms between the first and the second generation are strongly degenerated by the constraints of the experiments on $K - \overline{K}$ mixing and $\mu \to e\gamma$ decay. According to Ref.[9], the soft mass for $\tilde{D}$ and $\tilde{L}$ can receive the correction as $\mathcal{O}(F_\phi^2/M^4)$. In the present model for three neutrino case we have the relation $f_{e\mu} \gg f_{e\tau} \gg f_{\mu\tau}$. Due to this relation the corrections for first and second sleptons are common and the model is safe from the above constraints. For four neutrino models there exists new term $S10_f\overline{10^0_M}$, which gives the shifts of soft masses for $\tilde{\overline{E}}$, $\tilde{Q}$ and $\tilde{\overline{U}}$. Though the constraints from the experiments are rather severe under the relation $f_{e\mu} \ll f_{e\tau} < f_{\mu\tau}$, it is expected that the order of mass shift can be phenomenologically acceptable since $k_e = k_\mu \simeq 1$. This provides the suitable neutrino mass matrix as we argue in this paper.

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