

# Large Lepton Flavor Mixings in $SU(6) \times SU(2)_R$ Model

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## Abstract

The lepton masses and mixings are studied on the basis of string inspired  $SU(6) \times SU(2)_R$  model with global flavor symmetries. Provided that sizable mixings between lepton doublets  $L$  and Higgsino-like fields  $H_d$  with even  $R$ -parity occur and that seesaw mechanism is at work in the neutrino sector, the model can yield a large mixing angle solution with  $\tan \theta_{12}, \tan \theta_{23} = \mathcal{O}(\sqrt{\lambda})$  ( $\lambda \simeq 0.22$ ), which is consistent with the recent experimental data on atmospheric and solar neutrinos. In the solution Dirac mass hierarchies in the neutrino sector cancel out with the heavy Majorana sector in large part due to seesaw mechanism. Hierarchical pattern of charged lepton masses can be also explained.

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Recent experimental data have suggested large lepton flavor mixings. In fact, the latest atmospheric neutrino results from Super-Kamiokande are consistent with  $\nu_\mu$ - $\nu_\tau$  oscillation with  $\sin^2 2\theta_{23} > 0.88$  and  $\Delta m_{23}^2 = (1.5 \sim 5) \times 10^{-3} \text{eV}^2$ [1]. The latest solar neutrino results from Super-Kamiokande are in favor of the large mixing angle MSW region for  $\nu_e$ - $\nu_\mu$  oscillation with  $\sin^2 2\theta_{12} \sim 0.75$  and  $\Delta m_{12}^2 \sim 2.2 \times 10^{-5} \text{eV}^2$ [2]. These results indicate that lepton flavor mixing matrix (MNS matrix) is remarkably different from quark flavor mixing matrix (CKM matrix) in their hierarchical structure. At first sight it seems that the distinct flavor mixings of quarks and leptons are in disaccord with the quark-lepton unification. However, in a wide class of unification models, the situation is not so simple. This is because the massless sector in the supersymmetric unification theory includes extra particles beyond the standard model and then there may occur extra-particle mixings such as between quarks (leptons) and colored Higgsino-like fields (doublet Higgsino-like fields) with even  $R$ -parity. In order to study fermion masses and mixings we have to take into account the effects of such extra-particle mixings. In addition, in the neutrino sector we should incorporate the extra-particle mixings with seesaw mechanism [3]. In the previous paper[4] we have shown that the observed hierarchical structure of quark masses and CKM matrix can be naturally understood in  $SU(6) \times SU(2)_R$  model. In short, after integrating out heavy modes which get masses at intermediate energy scales, we have found that the extra-particle mixings possibly cause Yukawa hierarchies to change significantly. In this paper we study characteristic features of lepton masses and MNS matrix in the context of the string inspired  $SU(6) \times SU(2)_R$  model with global flavor symmetries.

The model discussed here is the same as in Ref.[4, 5, 6, 7]. Here we enumerate main points of the present model.

- (i). We choose  $SU(6) \times SU(2)_R$  as the unification gauge symmetry at the string scale  $M_S$ , which can be derived from the perturbative heterotic superstring theory via the flux breaking[8].
- (ii). Matter superfields consist of three family and one vector-like multiplet, i.e.,

$$3 \times \mathbf{27}(\Phi_{1,2,3}) + (\mathbf{27}(\Phi_0) + \overline{\mathbf{27}}(\overline{\Phi})) \quad (1)$$

in terms of  $E_6$ . Under  $G = SU(6) \times SU(2)_R$ , the superfields  $\Phi$  in  $\mathbf{27}$  of  $E_6$  are decomposed into two groups as

$$\Phi(\mathbf{27}) = \begin{cases} \phi(\mathbf{15}, \mathbf{1}) & : \quad Q, L, g, g^c, S, \\ \psi(\overline{\mathbf{6}}, \mathbf{2}) & : \quad (U^c, D^c), (N^c, E^c), (H_u, H_d), \end{cases} \quad (2)$$

where  $g, g^c$  and  $H_u, H_d$  represent colored Higgs and doublet Higgs fields, respectively.  $N^c$  is the right-handed neutrino superfield and  $S$  is an  $SO(10)$ -singlet. It is

noticeable that under  $G$  doublet Higgs and color-triplet Higgs fields belong to different representations. This situation is favorable to solve the triplet-doublet splitting problem. Although  $D^c$  and  $g^c(L$  and  $H_d)$  have the same quantum numbers under the standard model gauge group  $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ , they belong to different irreducible representations of  $G$ .

- (iii). We assign odd  $R$ -parity for  $\Phi_{1,2,3}$  and even for  $\Phi_0$  and  $\bar{\Phi}$ , respectively. Since ordinary Higgs doublets have even  $R$ -parity, they belong to  $\Phi_0$ . It is assumed that  $R$ -parity remains unbroken down to the electroweak scale.
- (iv). The gauge symmetry  $G$  is assumed to be spontaneously broken at  $|\langle\phi_0(\mathbf{15}, \mathbf{1})\rangle|$  and subsequently at  $|\langle\psi_0(\bar{\mathbf{6}}, \mathbf{2})\rangle|$ . Since the fields which develop non-zero VEV's are singlets under the remaining gauge symmetries, they are assigned as  $\langle\phi_0(\mathbf{15}, \mathbf{1})\rangle = \langle S_0 \rangle$  and  $\langle\psi_0(\bar{\mathbf{6}}, \mathbf{2})\rangle = \langle N_0^c \rangle$ . The  $D$ -flat conditions require  $\langle S_0 \rangle = \langle \bar{S} \rangle$  and  $\langle N_0^c \rangle = \langle \bar{N}^c \rangle$  at each step of the symmetry breakings

$$G = SU(6) \times SU(2)_R \xrightarrow{\langle S_0 \rangle} SU(4)_{\text{PS}} \times SU(2)_L \times SU(2)_R \xrightarrow{\langle N_0^c \rangle} G_{SM}, \quad (3)$$

where  $SU(4)_{\text{PS}}$  represents the Pati-Salam  $SU(4)$ [9]. Hereafter it is supposed that the symmetry breaking scales are roughly  $\langle S_0 \rangle = 10^{17\sim 18}\text{GeV}$  and  $\langle N_0^c \rangle = 10^{15\sim 17}\text{GeV}$ . In the present model the symmetry breakings at such large scales can be realized[10]. At the first step of the symmetry breaking fields  $Q_0$ ,  $L_0$ ,  $\bar{Q}$ ,  $\bar{L}$  and  $(S_0 - \bar{S})/\sqrt{2}$  are absorbed by gauge fields. Through the subsequent symmetry breaking fields  $U_0^c$ ,  $E_0^c$ ,  $\bar{U}^c$ ,  $\bar{E}^c$  and  $(N_0^c - \bar{N}^c)/\sqrt{2}$  are absorbed.

- (v). Gauge invariant trilinear couplings in the superpotential  $W$  become to be of the forms

$$(\phi(\mathbf{15}, \mathbf{1}))^3 = QQg + Qg^cL + g^c gS, \quad (4)$$

$$\begin{aligned} \phi(\mathbf{15}, \mathbf{1})(\psi(\bar{\mathbf{6}}, \mathbf{2}))^2 &= QH_dD^c + QH_uU^c + LH_dE^c + LH_uN^c \\ &\quad + SH_uH_d + gN^cD^c + gE^cU^c + g^cU^cD^c. \end{aligned} \quad (5)$$

- (vi). We introduce a global flavor symmetry  $\mathbf{Z}_N$ . Then the Froggatt-Nielsen mechanism is at work for the interactions[11]. The string theory naturally provides the discrete symmetry stemming from the symmetric structure of the compactified space. The stringy discrete symmetry may be either  $R$  symmetry or non- $R$  symmetry. Here we take a non- $R$  discrete symmetry.

In the present framework the effective Yukawa interactions for charged leptons are of the form[5]

$$M_{ij} L_i E_j^c H_{d0} \quad (6)$$

with

$$M_{ij} = (M_0)_{ij} \left( \frac{\langle X \rangle}{M_S} \right)^{e_{ij}} = m_{ij} x^{e_{ij}}, \quad (7)$$

where the subscripts  $i$  and  $j$  stand for the generation indices and the coupling constants  $m_{ij}$ 's are assumed to be  $\mathcal{O}(1)$  with  $\text{rank } m_{ij} = 3$ .  $X \equiv (S_0 \bar{S})/M_S$  is a singlet with a nonzero flavor charge and  $x \equiv \langle X \rangle/M_S < 1$ . The exponents  $e_{ij}$  are some non-negative integers which are settled by the flavor symmetry. Yukawa hierarchies are derived by assigning appropriate flavor charges to the matter fields. Concretely, when  $a_i, b_i$  ( $i = 0, 1, 2, 3$ ) and  $\bar{a}, \bar{b}$  denote the flavor charges of matter fields  $\phi_i, \psi_i$  and  $\bar{\phi}, \bar{\psi}$ , respectively, the singlet  $X$  has its charge  $a_X = a_0 + \bar{a}$ . Provided that  $|a_X|$  and  $N$  are prime with each other, we can take  $a_X = -1$  without loss of generality. In this case the flavor symmetry yields the relation

$$e_{ij} = a_i + b_j + b_0 \quad \text{mod } N \quad (8)$$

for the above effective Yukawa interactions. Hereafter we use the notation  $\alpha_i$  and  $\beta_i$  ( $i = 1, 2, 3$ ) defined by

$$\alpha_i = a_i - a_3, \quad \beta_i = b_i - b_3. \quad (9)$$

By definition we have  $\alpha_3 = \beta_3 = 0$ . Assuming

$$e_{33} = a_3 + b_3 + b_0 = 0 \quad \text{mod } N, \quad (10)$$

we have a  $3 \times 3$  mass matrix

$$M = \begin{pmatrix} m_{11}x^{\alpha_1+\beta_1} & m_{12}x^{\alpha_1+\beta_2} & m_{13}x^{\alpha_1} \\ m_{21}x^{\alpha_2+\beta_1} & m_{22}x^{\alpha_2+\beta_2} & m_{23}x^{\alpha_2} \\ m_{31}x^{\beta_1} & m_{32}x^{\beta_2} & m_{33} \end{pmatrix}. \quad (11)$$

By virtue of  $SU(6) \times SU(2)_R$  gauge symmetry the up-type quark mass matrix is given by the same matrix  $M$ . The assumption  $e_{33} = 0$  implies that the Yukawa coupling for top-quark is  $\mathcal{O}(1)$ . In the previous paper[4] we showed that hierarchical pattern of quark masses and mixings can be reproduced by taking

$$x^{\alpha_1} = \lambda^3, \quad x^{\alpha_2} = \lambda^2, \quad x^{\beta_1} = \lambda^4, \quad x^{\beta_2} = \lambda^2 \quad (12)$$

with  $\lambda \simeq 0.22$ . Hereafter we take this choice of the parameters.

Below the scale  $\langle N_0^c \rangle$  there appear both  $L$ - $H_d$  and  $D^c$ - $g^c$  mixings. Due to  $L$ - $H_d$  mixings

the charged lepton mass matrix is expressed in terms of the  $6 \times 6$  matrix

$$\widehat{M}_l = \begin{matrix} & H_u^+ & E^{c+} \\ H_d^- & \begin{pmatrix} y_S H & 0 \\ y_N M & \rho_d M \end{pmatrix} \\ L^- & \end{matrix} \quad (13)$$

in units of the string scale  $M_S$ . Three nonzero  $3 \times 3$  matrices arise from the mass terms  $H_{ij} H_{di} H_{uj} \langle S_0 \rangle$ ,  $M_{ij} L_i H_{uj} \langle N_0^c \rangle$  and  $M_{ij} L_i E_j^c \langle H_{d0} \rangle$ , where

$$H_{ij} = (H_0)_{ij} x^{\beta_i + \beta_j + \xi} = h_{ij} x^{\beta_i + \beta_j + \xi} \quad (14)$$

with  $h_{ij} = \mathcal{O}(1)$ . The exponent  $\xi$  represents the flavor charge of the trilinear products  $H_{d3} H_{u3} S_0$ , i.e.,  $\xi = 2b_3 + a_0$ . Here  $\xi$  is taken to be non-negative so that the trilinear couplings  $H_{ij} H_{di} H_{uj} S_0$  have the Yukawa hierarchy  $|H_{1j}| \ll |H_{2j}| \ll |H_{3j}|$  for  $j = 1, 2, 3$  similar to  $M_{ij}$ . Here we use the notations  $y_S$ ,  $y_N$  and  $\rho_d$  for the VEV's  $\langle S_0 \rangle$ ,  $\langle N_0^c \rangle$  and  $\langle H_{d0} \rangle = v_d$  in  $M_S$  units, respectively. From Eq.(5) it is found that the matrix  $H$  is symmetric. Since  $\rho_d$  is very small compared to  $y_S$  and  $y_N$ , the mixings between  $E^c$  and  $H_u$  are negligibly small. While the large mixings between  $L$  and  $H_d$  can occur depending on the relative magnitude of  $y_S H$  and  $y_N M$ .

The matrix  $\widehat{M}_l$  can be diagonalized by a bi-unitary transformation as

$$\widehat{\mathcal{V}}_l^{-1} \widehat{M}_l \widehat{\mathcal{U}}_l. \quad (15)$$

To solve the eigenvalue problem, it is more instructive for us to take  $\widehat{M}_l^\dagger \widehat{M}_l$  expressed as

$$\widehat{M}_l^\dagger \widehat{M}_l = \begin{pmatrix} A_l + B_l & \epsilon_d B_l \\ \epsilon_d B_l & \epsilon_d^2 B_l \end{pmatrix}, \quad (16)$$

where  $A_l = y_S^2 H^\dagger H$  and  $B_l = y_N^2 M^\dagger M$  with  $\epsilon_d \equiv \rho_d / y_N$ . Since  $\epsilon_d$  is a very small number, we can carry out our calculation by using perturbative  $\epsilon_d$ -expansion. Among six eigenvalues three of them are given by eigenvalues of  $(A_l + B_l)$  at the leading order, which represent heavy modes with the GUT scale masses. The remaining three are derived from diagonalization of  $\epsilon_d^2$ -terms, i.e.,

$$\epsilon_d^2 B_l - \epsilon_d B_l \frac{1}{A_l + B_l} \epsilon_d B_l = \epsilon_d^2 (A_l^{-1} + B_l^{-1})^{-1}. \quad (17)$$

These small eigenvalues correspond to masses squared of charged leptons ( $e, \mu, \tau$ ). Unitary transformations which diagonalize  $(A_l + B_l)$  and  $\epsilon_d^2 (A_l^{-1} + B_l^{-1})^{-1}$  are written as  $\mathcal{W}_l$  and  $\mathcal{V}_l$ , namely

$$\mathcal{W}_l^{-1} (A_l + B_l) \mathcal{W}_l = (\Lambda_l^{(0)})^2, \quad (18)$$

$$\epsilon_d^2 \mathcal{V}_l^{-1} (A_l^{-1} + B_l^{-1})^{-1} \mathcal{V}_l = \epsilon_d^2 (\Lambda_l^{(2)})^2, \quad (19)$$

where  $\Lambda_l^{(0)}$  and  $\Lambda_l^{(2)}$  are diagonal. Thus explicit forms of the unitary matrices  $\widehat{\mathcal{V}}_l$  and  $\widehat{\mathcal{U}}_l$  in Eq.(15) are

$$\widehat{\mathcal{V}}_l \simeq \begin{pmatrix} y_S H \mathcal{W}_l \Lambda_l^{(0)-1} & -y_S^{-1} H^{\dagger-1} \mathcal{V}_l \Lambda_l^{(2)} \\ y_N M \mathcal{W}_l \Lambda_l^{(0)-1} & y_N^{-1} M^{\dagger-1} \mathcal{V}_l \Lambda_l^{(2)} \end{pmatrix}, \quad (20)$$

$$\widehat{\mathcal{U}}_l \simeq \begin{pmatrix} \mathcal{W}_l & -\epsilon_d (A_l + B_l)^{-1} B_l \mathcal{V}_l \\ \epsilon_d B_l (A_l + B_l)^{-1} \mathcal{W}_l & \mathcal{V}_l \end{pmatrix} \quad (21)$$

in the  $\epsilon_d$  expansion. From Eq. (20), the mass eigenstates of light  $SU(2)_L$ -doublet charged leptons are given by

$$|\tilde{L}^-\rangle = \Lambda_l^{(2)} \mathcal{V}_l^T \left( -y_S^{-1} H^{*-1} |H_d^-\rangle + y_N^{-1} M^{*-1} |L^-\rangle \right). \quad (22)$$

Consequently, provided that the elements of  $y_S^{-1} H^{*-1}$  and  $y_N^{-1} M^{*-1}$  are comparable to each other, there occur large mixings between  $H_d^-$  and  $L^-$ . In order to parametrize the relative magnitude of  $L^-$ - and  $H_d^-$ -components we introduce the notation

$$r_l = \frac{y_S}{y_N} x^\xi = \frac{\langle S_0 \rangle}{\langle N_0^c \rangle} x^\xi \sim \frac{y_S H_{33}}{y_N M_{33}}. \quad (23)$$

We now proceed to calculate the eigenvalues of  $\epsilon_d^2 (A_l^{-1} + B_l^{-1})^{-1}$ . Generally, when a  $3 \times 3$  Hermite matrix  $C$  has hierarchical pattern as shown in Eq.(11), three eigenvalues of the matrix  $C$  are approximately expressed as

$$\text{Tr}(C), \quad \frac{\sum_i \Delta(C)_{ii}}{\text{Tr}(C)}, \quad \frac{\det C}{\sum_i \Delta(C)_{ii}}, \quad (24)$$

where  $\Delta(C)_{ij}$  represents the cofactor for the  $(i, j)$  element of  $C$ . When applied to  $(A_l^{-1} + B_l^{-1})$ , hierarchies of the eigenvalues depend on the relative magnitude of the elements of  $A_l^{-1}$  and  $B_l^{-1}$ , which are controlled by  $r_l$  as

$$(A_l^{-1} + B_l^{-1})_{ij} = y_N^{-2} x^{-(\beta_i + \beta_j)} \sum_k \{ r_l^{-2} x^{-2\beta_k} \bar{h}_{ki}^* \bar{h}_{kj} + x^{-2\alpha_k} \bar{m}_{ki}^* \bar{m}_{kj} \}, \quad (25)$$

where we denote  $\bar{h}_{ij} = (H_0^{\dagger-1})_{ij}$  and  $\bar{m}_{ij} = (M_0^{\dagger-1})_{ij}$ . Let us consider the following four regions of the parameter  $r_l$ , provided that the phenomenological conditions  $m_e \geq \mathcal{O}(\lambda^9 v_d)$  and  $m_\tau < \mathcal{O}(v_d)$  are satisfied.

**Case (i)**  $x^{\alpha_1-\beta_2}(=\lambda) \leq r_l < x^{\alpha_2-\beta_2}(=1)$

From Eq.(24) light charged lepton masses become

$$\begin{aligned} m_e &\sim r_l x^{2\beta_1} v_d = r_l \lambda^8 v_d, \\ m_\mu &\sim x^{\alpha_1+\beta_2} v_d = \lambda^5 v_d, \\ m_\tau &\sim r_l x^{\beta_2} v_d = r_l \lambda^2 v_d. \end{aligned} \quad (26)$$

**Case (ii)**  $x^{\alpha_2-\beta_2}(=1) \leq r_l < x^{\alpha_1-\beta_1}(=\lambda^{-1})$

Charged lepton masses become

$$\begin{aligned} m_e &\sim r_l x^{2\beta_1} v_d = r_l \lambda^8 v_d, \\ m_\mu &\sim x^{\alpha_1+\beta_2} v_d = \lambda^5 v_d, \\ m_\tau &\sim x^{\alpha_2} v_d = \lambda^2 v_d. \end{aligned} \quad (27)$$

**Case (iii)**  $x^{\alpha_1-\beta_1}(=\lambda^{-1}) \leq r_l < x^{\alpha_2-\beta_1}(=\lambda^{-2})$

In this region we obtain

$$\begin{aligned} m_e &\sim x^{\alpha_1+\beta_1} v_d = \lambda^7 v_d, \\ m_\mu &\sim r_l x^{\beta_1+\beta_2} v_d = r_l \lambda^6 v_d, \\ m_\tau &\sim x^{\alpha_2} v_d = \lambda^2 v_d. \end{aligned} \quad (28)$$

**Case (iv)**  $x^{\alpha_2-\beta_1}(=\lambda^{-2}) \leq r_l < x^{-\beta_1}(=\lambda^{-4})$

In this region we have

$$\begin{aligned} m_e &\sim x^{\alpha_1+\beta_1} v_d = \lambda^7 v_d, \\ m_\mu &\sim x^{\alpha_2+\beta_2} v_d = \lambda^4 v_d, \\ m_\tau &\sim r_l x^{\beta_1} v_d = r_l \lambda^4 v_d. \end{aligned} \quad (29)$$

Thus mass hierarchy of charged leptons apparently changes depending on the parameter  $r_l$ . Experimentally the hierarchical charged lepton masses are summarized as

$$\frac{m_e}{m_\mu} \simeq \lambda^{3.5} \quad \frac{m_\mu}{m_\tau} \simeq \lambda^2. \quad (30)$$

Among the above solutions the case (ii) gives rather large hierarchy  $m_\mu/m_\tau \simeq \lambda^3$  as given in Eq.(26). For other three cases we obtain reasonable hierarchies for  $m_e/m_\mu$  and  $m_\mu/m_\tau$  by adjusting  $r_l$  as

$$\begin{aligned} \frac{m_e}{m_\mu} &\simeq \lambda^{3.5} & \frac{m_\mu}{m_\tau} &\simeq \lambda^{2.5} & \text{for the case (i)} & (r_l \simeq \sqrt{\lambda}) \\ \frac{m_e}{m_\mu} &\simeq \lambda^3 & \frac{m_\mu}{m_\tau} &\simeq \lambda^2 & \text{for the cases (iii) and (iv)} & (r_l \simeq \lambda^{-2}). \end{aligned} \quad (31)$$

In the followings we investigate the neutral lepton sector to combine the charged lepton solutions with light neutrino ones.

In the neutral lepton sector we have the  $15 \times 15$  mass matrix

$$\widehat{M}_N = \begin{matrix} & H_u^0 & H_d^0 & L^0 & N^c & S \\ \begin{matrix} H_u^0 \\ H_d^0 \\ L^0 \\ N^c \\ S \end{matrix} & \left( \begin{array}{ccccc} 0 & y_S H & y_N M^T & 0 & \rho_d M^T \\ y_S H & 0 & 0 & 0 & \rho_u M^T \\ y_N M & 0 & 0 & \rho_u M & 0 \\ 0 & 0 & \rho_u M^T & N & T^T \\ \rho_d M & \rho_u M & 0 & T & S \end{array} \right) \end{matrix} \quad (32)$$

in  $M_S$  units, where  $\rho_u = \langle H_{u0} \rangle / M_S = v_u / M_S$ . The  $6 \times 6$  submatrix

$$\widehat{M}_M = \begin{pmatrix} N & T^T \\ T & S \end{pmatrix} \quad (33)$$

represents the Majorana mass terms which come from the nonrenormalizable interactions  $(\Phi_i \overline{\Phi})(\Phi_j \overline{\Phi})(\Phi_0 \overline{\Phi})^{l_{ij}}$  with non-negative integers  $l_{ij}$ . Since matter fields  $N_i^c$  and  $S_i$  reside in the multiplets  $\psi(\overline{\mathbf{6}}, \mathbf{2})_i$  and  $\phi(\mathbf{15}, \mathbf{1})_i$ , respectively, this matrix has hierarchical structure. If the magnitude of the Majorana mass terms is large enough compared to the electroweak scale, due to seesaw mechanism we can obtain small neutrino masses. By recalling the above study in the charged lepton sector, it is easy to see that the unitary matrix  $\widehat{U}_N$  which diagonalizes  $\widehat{M}_N$  can be approximately decomposed into three factors as

$$\widehat{U}_N = \widehat{U}_N^{(0)} \widehat{U}_N^{(1)} \widehat{U}_N^{(2)}, \quad (34)$$

where the matrix  $\widehat{U}_N^{(0)}$  is essentially the same as the diagonalization matrix for light charged leptons and

$$\widehat{U}_N^{(1)} \simeq \begin{pmatrix} I_{9 \times 9} & 0 \\ 0 & \widehat{U}_M \end{pmatrix}, \quad \widehat{U}_N^{(2)} \simeq \begin{pmatrix} I_{6 \times 6} & 0 & 0 \\ 0 & \mathcal{V} & 0 \\ 0 & 0 & I_{6 \times 6} \end{pmatrix}. \quad (35)$$

The matrix  $\widehat{U}_N^{(1)}$  means the diagonalization matrix for the Majorana mass matrix (33). The matrix  $\widehat{U}_N^{(2)}$  represents a diagonalization matrix for light neutrinos. It turns out that the light neutrino mass eigenstates are

$$|\tilde{L}^0\rangle = \mathcal{V}^T \Lambda_l^{(2)} \mathcal{V}_l^T \left( -y_S^{-1} H^{*-1} |H_d^0\rangle + y_N^{-1} M^{*-1} |L^0\rangle \right). \quad (36)$$



Comparing these eigenstates  $\tilde{L}^0$  with those of light charged leptons  $\tilde{L}^-$  given by Eq. (22), we find that  $\mathcal{V}$  is nothing but MNS matrix. The matrix  $\mathcal{V}$  represents an additional transformation for neutrinos on the mass-diagonal basis for light charged leptons.  $\mathcal{V}$  is determined as the diagonalization matrix for the neutrino mass matrix  $M_\nu$  defined by

$$M_\nu = M_S \epsilon_u^2 \left( \Lambda_l^{(2)} \mathcal{V}_l^{-1} R^{-1} \mathcal{V}_l^* \Lambda_l^{(2)} \right), \quad (37)$$

where  $\epsilon_u = v_u / \langle N_0^c \rangle$  and  $R$  is the induced Majorana mass matrix stemming from Eq.(33). Note that the matrix  $R$  has the hierarchical structure given by

$$R = y_R \begin{pmatrix} r_{11} x^{2\beta_1} & r_{12} x^{\beta_1 + \beta_2} & r_{13} x^{\beta_1} \\ r_{21} x^{\beta_1 + \beta_2} & r_{22} x^{2\beta_2} & r_{23} x^{\beta_2} \\ r_{31} x^{\beta_1} & r_{32} x^{\beta_2} & r_{33} \end{pmatrix} \quad (38)$$

with symmetric  $\mathcal{O}(1)$  numbers  $r_{ij}$  and  $M_S y_R$  represents the Majorana mass scale. As seen from Eq.(37) Dirac mass hierarchies given by  $\Lambda_l^{(2)}$  cancel out in part or in large part due to seesaw mechanism. Since Dirac mass hierarchies depend on the parameter  $r_l$ , the neutrino mass matrix also depends on  $r_l$ . Thus we consider the following four cases separately.

**Case (i)**  $x^{\alpha_1 - \beta_2} (= \lambda) \leq r_l < x^{\alpha_2 - \beta_2} (= 1)$

In this case the neutrino mass matrix becomes

$$M_\nu = \frac{v_u^2}{M_S y_R} \times \begin{pmatrix} O(r_l^2 x^{2\beta_1}) & O(r_l x^{\alpha_1 + \beta_1}) & O(r_l^2 x^{\beta_1 + \beta_2}) \\ O(r_l x^{\alpha_1 + \beta_1}) & O(x^{2\alpha_1}) & O(r_l x^{\alpha_1 + \beta_2}) \\ O(r_l^2 x^{\beta_1 + \beta_2}) & O(r_l x^{\alpha_1 + \beta_2}) & O(r_l^2 x^{2\beta_2}) \end{pmatrix}. \quad (39)$$

This leads to the mixing angles in MNS matrix

$$\begin{aligned} \tan \theta_{12} &\sim r_l x^{\beta_1 - \alpha_1} = r_l \lambda, \\ \tan \theta_{23} &\sim r_l^{-1} x^{\alpha_1 - \beta_2} = r_l^{-1} \lambda, \\ \tan \theta_{13} &\sim x^{\beta_1 - \beta_2} = \lambda^2, \end{aligned} \quad (40)$$

where  $\theta_{ij}$ 's are defined as

$$U_{MNS} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} & c_{12} c_{23} - s_{12} s_{23} s_{13} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} & -c_{12} s_{23} - s_{12} c_{23} s_{13} & c_{23} c_{13} \end{pmatrix}. \quad (41)$$

The ratios of neutrino masses are given by

$$m_1 : m_2 : m_3 \sim r_l^2 x^{2\beta_1} : x^{2\alpha_1} : r_l^2 x^{2\beta_2} = r_l^2 \lambda^8 : \lambda^6 : r_l^2 \lambda^4. \quad (42)$$

**Case (ii)**  $x^{\alpha_2-\beta_2}(=1) \leq r_l < x^{\alpha_1-\beta_1}(=\lambda^{-1})$

In this case the neutrino mass matrix becomes

$$M_\nu = \frac{v_u^2}{M_S y_R} \times \begin{pmatrix} O(r_l^2 x^{2\beta_1}) & O(r_l x^{\alpha_1+\beta_1}) & O(r_l x^{\alpha_2+\beta_1}) \\ O(r_l x^{\alpha_1+\beta_1}) & O(x^{2\alpha_1}) & O(x^{\alpha_1+\alpha_2}) \\ O(r_l x^{\alpha_2+\beta_1}) & O(x^{\alpha_1+\alpha_2}) & O(x^{2\alpha_2}) \end{pmatrix}. \quad (43)$$

This leads to the mixing angles in MNS matrix

$$\begin{aligned} \tan \theta_{12} &\sim r_l x^{\beta_1-\alpha_1} = r_l \lambda, \\ \tan \theta_{23} &\sim x^{\alpha_1-\alpha_2} = \lambda, \\ \tan \theta_{13} &\sim r_l x^{\beta_1-\alpha_2} = r_l \lambda^2. \end{aligned} \quad (44)$$

The ratios of neutrino masses are given by

$$m_1 : m_2 : m_3 \sim r_l^2 x^{2\beta_1} : x^{2\alpha_1} : x^{2\alpha_2} = r_l^2 \lambda^8 : \lambda^6 : \lambda^4. \quad (45)$$

**Case (iii)**  $x^{\alpha_1-\beta_1}(=\lambda^{-1}) \leq r_l < x^{\alpha_2-\beta_1}(=\lambda^{-2})$

In this case the neutrino mass matrix becomes

$$M_\nu = \frac{v_u^2}{M_S y_R} \times \begin{pmatrix} O(x^{2\alpha_1}) & O(r_l x^{\alpha_1+\beta_1}) & O(x^{\alpha_1+\alpha_2}) \\ O(r_l x^{\alpha_1+\beta_1}) & O(r_l^2 x^{2\beta_1}) & O(r_l x^{\alpha_2+\beta_1}) \\ O(x^{\alpha_1+\alpha_2}) & O(r_l x^{\alpha_2+\beta_1}) & O(x^{2\alpha_2}) \end{pmatrix}. \quad (46)$$

We obtain the mixing angles

$$\begin{aligned} \tan \theta_{12} &\sim r_l^{-1} x^{\alpha_1-\beta_1} = (r_l \lambda)^{-1}, \\ \tan \theta_{23} &\sim r_l x^{\beta_1-\alpha_2} = r_l \lambda^2, \\ \tan \theta_{13} &\sim x^{\alpha_1-\alpha_2} = \lambda. \end{aligned} \quad (47)$$

The ratios of neutrino masses are

$$m_1 : m_2 : m_3 \sim x^{2\alpha_1} : r_l^2 x^{2\beta_1} : x^{2\alpha_2} = \lambda^6 : r_l^2 \lambda^8 : \lambda^4. \quad (48)$$

**Case (iv)**  $x^{\alpha_2-\beta_1}(=\lambda^{-2}) \leq r_l < x^{-\beta_1}(=\lambda^{-4})$

In this case the neutrino mass matrix becomes

$$M_\nu = \frac{v_u^2}{M_S y_R} \times \begin{pmatrix} O(x^{2\alpha_1}) & O(x^{\alpha_1+\alpha_2}) & O(r_l x^{\alpha_1+\beta_1}) \\ O(x^{\alpha_1+\alpha_2}) & O(x^{2\alpha_2}) & O(r_l x^{\alpha_2+\beta_1}) \\ O(r_l x^{\alpha_1+\beta_1}) & O(r_l x^{\alpha_2+\beta_1}) & O(r_l^2 x^{2\beta_1}) \end{pmatrix}. \quad (49)$$

We have the mixing angles

$$\begin{aligned}\tan \theta_{12} &\sim x^{\alpha_1 - \alpha_2} = \lambda, \\ \tan \theta_{23} &\sim r_l^{-1} x^{\alpha_2 - \beta_1} = (r_l \lambda^2)^{-1}, \\ \tan \theta_{13} &\sim r_l^{-1} x^{\alpha_1 - \beta_1} = (r_l \lambda)^{-1}.\end{aligned}\tag{50}$$

The ratios of neutrino masses are

$$m_1 : m_2 : m_3 \sim x^{2\alpha_1} : x^{2\alpha_2} : r_l^2 x^{2\beta_1} = \lambda^6 : \lambda^4 : r_l^2 \lambda^8.\tag{51}$$

As mentioned above, the characteristic pattern of neutrino masses and mixing angles varies significantly depending on the parameter  $r_l$ . As seen from Eq.(23), the choice of  $r_l$  corresponds to the adjustment of  $\xi (= 2b_3 + a_0)$ . A large mixing angle solution in which both  $\tan \theta_{12}$  and  $\tan \theta_{23}$  are  $\mathcal{O}(1)$  can be realized by taking  $r_l \sim \lambda^{-1.5}$  in the case (iii). Otherwise, at least one of  $\tan \theta_{12}$  and  $\tan \theta_{23}$  becomes small. In Ref.[6] we adopted the value of  $r_l = 1$  in the case (i) or (ii) and then we obtained slightly small mixing angles, i.e.,  $\tan \theta_{12}, \tan \theta_{23} = \mathcal{O}(\lambda)$ . Instead we choose

$$r_l = \lambda^{-1.5}\tag{52}$$

in the case (iii). This choice of  $r_l$  together with  $e_{33} = 0$  exhibits constraints on the flavor charge assignment for matter fields. It should be emphasized that in the case (iii) Dirac mass hierarchies cancel out with  $R^{-1}$  in large part due to seesaw mechanism. Thus we have the lepton mass hierarchies

$$m_e \sim \lambda^7 v_d, \quad m_\mu \sim \lambda^{4.5} v_d, \quad m_\tau \sim \lambda^2 v_d,\tag{53}$$

$$m_1 \sim \lambda^6 \frac{v_u^2}{M_S y_R}, \quad m_2 \sim \lambda^5 \frac{v_u^2}{M_S y_R}, \quad m_3 \sim \lambda^4 \frac{v_u^2}{M_S y_R},\tag{54}$$

which lead to the numerical values

$$m_e \sim 2 \text{ MeV}, \quad m_\mu \sim 100 \text{ MeV}, \quad m_\tau \sim 5 \text{ GeV},\tag{55}$$

$$m_1 \sim 0.002 \text{ eV}, \quad m_2 \sim 0.01 \text{ eV}, \quad m_3 \sim 0.05 \text{ eV},\tag{56}$$

provided that  $v_u, v_d \sim 100 \text{ GeV}$  and  $M_S y_R = 5 \times 10^{11} \text{ GeV}$ . Further, the mixing angles become

$$\tan \theta_{12} \sim \sqrt{\lambda}, \quad \tan \theta_{23} \sim \sqrt{\lambda}, \quad \tan \theta_{13} \sim \lambda.\tag{57}$$

If  $\tan \theta = \sqrt{\lambda} = 0.47$ , then we obtain  $\sin^2 2\theta = 0.59$ . Since Eq.(57) is an order of magnitude relationship, the above results are consistent with the recent data on atmospheric and solar neutrinos. As for neutrino mass differences, we have the ratio

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \sim \frac{m_2^2}{m_3^2} \sim \lambda^2 \sim \frac{1}{20}.\tag{58}$$

This is also consistent with the data. In this model flavor mixings come from mainly the neutrino mass matrix because the charged lepton one is hierarchical and  $\mathcal{V}_l \sim 1$ , which is similar to the diagonalization matrix for quark mass matrices[4]. One of the characteristic features of the mixing matrix obtained here is to give a relatively large value for  $U_{e3}$ , which is  $\sim \lambda$  near to the experimental bound of CHOOZ( $\leq 0.16$ )[12], in contrast to  $V_{CKM,td}(\sim \lambda^3)$  or  $V_{CKM,ub}(\sim \lambda^4)$  in the quark mixing matrix. The bi-maximal mixing solution usually gives the tiny magnitude for  $U_{e3}$ [13]. The measurement of  $U_{e3}$  will give an important clue to distinguish the various models. The reactor experiment, for example, KamLAND[14] will cover the LMA-MSW region and possibly give the restriction on  $U_{e3}$  through  $\nu_e$  disappearance experiment as

$$\sum_{x=\mu,\tau} P(\nu_e \rightarrow \nu_x) = 4|U_{e3}|^2(1 - |U_{e3}|^2) \sin^2 \frac{\Delta m_{atm}^2 L}{4E},$$

$$\sin^2 2\theta_{reac} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \simeq 0.09 \quad (\text{for } |U_{e3}| \simeq 0.15). \quad (59)$$

As another choice of the parameter  $r_l$  it is interesting for us to take  $r_l \sim \sqrt{\lambda}$  in the case (i). In this choice lepton masses become

$$m_e \sim \lambda^{8.5} v_d, \quad m_\mu \sim \lambda^5 v_d, \quad m_\tau \sim \lambda^{2.5} v_d, \quad (60)$$

$$m_1 \sim \lambda^9 \frac{v_u^2}{M_S y_R}, \quad m_2 \sim \lambda^6 \frac{v_u^2}{M_S y_R}, \quad m_3 \sim \lambda^5 \frac{v_u^2}{M_S y_R}. \quad (61)$$

Numerically by taking  $M_S y_R = 2 \times 10^{12} \text{GeV}$  we obtain

$$m_e \sim 0.3 \text{ MeV}, \quad m_\mu \sim 50 \text{ MeV}, \quad m_\tau \sim 2 \text{ GeV}, \quad (62)$$

$$m_1 \sim 0.0001 \text{ eV}, \quad m_2 \sim 0.01 \text{ eV}, \quad m_3 \sim 0.05 \text{ eV}. \quad (63)$$

Further, we have the mixing angles

$$\tan \theta_{12} \sim \lambda^{1.5}, \quad \tan \theta_{23} \sim \sqrt{\lambda}, \quad \tan \theta_{13} \sim \lambda^2. \quad (64)$$

The magnitudes of  $\tan \theta_{23}$  and  $\Delta m_{12}^2 / \Delta m_{23}^2$  are the same as in the above solution with  $r_l = \lambda^{-1.5}$ . While the magnitude of  $\tan \theta_{12}$  is near the small mixing angle MSW region for  $\nu_e - \nu_\mu$  oscillation.

In conclusion, the present model can yield a large mixing angle solution with  $\tan \theta_{12}$ ,  $\tan \theta_{23} = \mathcal{O}(\sqrt{\lambda})$  together with lepton mass hierarchies. Dirac mass hierarchies in the neutrino sector cancel out with the heavy Majorana sector in large part due to seesaw mechanism. Hierarchical pattern of charged leptons is also explained. In the string inspired models the massless sector contains extra particles beyond the minimal supersymmetric standard model. In the course of the gauge symmetry breakings many particles become

massive or are absorbed by gauge fields via Higgs mechanism at intermediate energy scales. Therefore, after integrating out these heavy modes we derive the low-energy effective theory in which large extra-particle mixings cause an apparent change of the Yukawa hierarchies for leptons and down-type quarks. In the neutrino sector seesaw mechanism is also incorporated. This is the reason why nontrivial patterns appear in fermion mixing angles. Finally, we comment on the flavor symmetry. In the present model we need an appropriate discrete flavor symmetry and also the adjustment of the flavor charge assignment for matter fields. In our study we assign appropriate flavor charges to matter fields by hand so as to obtain an interesting solution. In the framework of string theory it is important for us to explore the selection rule including the flavor symmetry.

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