

Spontaneous CP Violation and Higgs Masses in the Next-to-Minimal Supersymmetric Model

Naoyuki HABA^{a,1}, Masahisa MATSUDA^{b,2} and Morimitsu TANIMOTO^{c,3}

^a*Department of Physics, Nagoya University
Nagoya, JAPAN 464-01*

^b*Department of Physics and Astronomy
Aichi University of Education
Kariya, Aichi, JAPAN 448*

^c*Science Education Laboratory, Ehime University,
Matsuyama, JAPAN 790*

Abstract

We study the possibility of spontaneous CP violation in the next-to-minimal supersymmetric standard model (NMSSM). It is shown that the spontaneous CP violation is induced by the radiative effects of top, stop, bottom and sbottom superfields. The available regions of parameters, which are obtained by imposing the constraints from experiments, are rather narrow. We also obtain strong constraints for light Higgs masses such as $m_H \leq 36\text{GeV}$ numerically. Sum of masses of two light neutral Higgs should set around 93GeV and charged Higgs boson has a rather higher mass larger than 700GeV .

PACS numbers:11.30.Er, 11.30.Qc, 12.60.Jv, 14.80.Cp

¹E-mail: haba@eken.phys.nagoya-u.ac.jp

²E-mail: mmatsuda@aecc.aichi-edu.ac.jp

³E-mail: tanimoto@edserv.ed.ehime-u.ac.jp

1 Introduction

The physics of CP violation has attracted much attention in the light that the B -factory will go on line in the near future at KEK and SLAC. The central subject of the B -factory is the test of the standard model(SM), in which the origin of CP violation is reduced to the phase in the Kobayashi-Maskawa matrix[1]. However, there has been a general interest in considering other approaches to CP violation since many alternate sources exist. The attractive extension of the standard Higgs sector is the two Higgs doublet model(THDM)[2], yielding both charged and neutral Higgs bosons as physical states. The THDM with the soft breaking term of the discrete symmetry demonstrates explicit or spontaneous CP violation [3][4][5]. On the other hand, the recent measurements of gauge couplings at M_Z scale suggest the minimal supersymmetric extension of the standard model(MSSM) is a good candidate beyond the standard model in the standpoint of the gauge unification[6].

It is well known that CP symmetry could be violated explicitly or spontaneously in the THDM without supersymmetry[7]. Though the MSSM contains two Higgs doublets $H_1^T = (H_1^0, H_1^-)$ and $H_2^T = (H_2^+, H_2^0)$, which give masses to down-quarks and up-quarks, respectively, there is no degree of freedom for CP violation at tree level Higgs potential. Spontaneous symmetry breaking of $SU(2)_L$ by taking non-zero real vacuum expectation values(VEV) gives rise to two CP -even neutral Higgs scalars, a CP -odd neutral pseudoscalar boson, and two charged Higgs bosons. One of two CP -even bosons is the lightest of all Higgs bosons in the MSSM and its tree level mass is less than that of Z^0 . However, large radiative corrections proportional to $(g^2 m_t^4 / M_W^2)$ increase the lightest mass of the neutral Higgs bosons of the MSSM than M_Z [8]. Within a framework of the MSSM it is also possible to violate CP symmetry spontaneously by radiative effects of heavy quarks with relatively non-zero complex VEVs for H_1^0 and H_2^0 [9]. Phenomenologically this model requires the lightest mass of the neutral Higgs boson to be a few GeV as a result of Geogi-Pais theorem[10]. So this interesting scenario to violate CP symmetry spontaneously in the MSSM is unfortunately inconsistent with the experiment which suggests that the lightest pseudoscalar Higgs mass is larger than 22GeV[11]. To avoid this difficulty, the simple extension of the MSSM has been considered to obtain explicit[12] or spontaneous[13] CP violation in the Higgs sector. The extension is that a singlet superfield

under $G_{st} = SU(3)_C \times SU(2)_L \times U(1)_Y$ is added to the MSSM. This model is usually called as next-to-minimal supersymmetric standard model(NMSSM) [14][15].

The NMSSM is introduced to solve so called μ -problem. The superpotential needs the term like $\mu H_1 H_2$ to give the non-zero VEVs for both Higgs doublets in the MSSM, where μ should be $O(M_W)$. However, the MSSM does not explain why μ should be so small. In the NMSSM we can introduce $\lambda N H_1 H_2$ -term in the superpotential, where N is a singlet superfield under G_{st} and λ is Higgs coupling with $O(\lambda) \simeq 1$. If N develops a non-zero VEV $\langle N \rangle \equiv x$, the μ -term is generated as $\mu = \lambda x \simeq O(M_W)$. Such a singlet field appears in grand unified supersymmetric models[16] and in massless sectors of superstring models[17] as well as in superstring models based on E_6 [18] and $SU(5) \times U(1)$ gauge groups[19]. The minimal extension of the MSSM with an additional singlet superfield is an attractive alternative and these models are analyzed by many authors with no spontaneous CP violation[20][21].

In the NMSSM, candidates to have non-zero VEVs are H_1^0, H_2^0 and N and it is likely to develop relatively complex VEVs to violate CP symmetry spontaneously. Furthermore, in order to obtain relatively large mass of neutral Higgs, x should be rather large compared to $v = \sqrt{v_1^2 + v_2^2} = 174\text{GeV}$, where v_1 and v_2 are the VEVs of H_1^0 and H_2^0 , respectively.

Recently Babu and Barr have shown that there exists the solution to lift the lightest mass of Higgs boson to the consistent region with the present experimental lower bound of its mass[13]. In this analysis they pointed out that the spontaneous CP violation occurred by the radiative effect of stop and top loop in the NMSSM for the parameter $\tan \beta = v_2/v_1 \simeq 1$. However, they used the simplified squark mass matrix as $m_{\tilde{t}_L} = m_{\tilde{t}_R}$ and neglected the sbottom and bottom contributions for one-loop correction. Furthermore there is one problem that the charged Higgs mass would be around 100GeV, which might be excluded in the minimal supergravity model[23] with the experiment $b \rightarrow s + \gamma$ [22]. The charged Higgs mass should be larger than 160GeV in this model for small $\tan \beta$, while it's limit is 250GeV in the THDM[24].

In this paper we introduce the full radiative corrections from top, stop, bottom and sbottom contribution in the NMSSM which derive the different results from Ref.[13]. We also determine the available parameter regions in the NMSSM with spontaneous CP -violation by imposing precise experimental constraints for the lower limit of neutral Higgs mass h from $Z \rightarrow h_1 + h_2$

and $Z \rightarrow h_1 + l^+ l^-$ decay processes. In particular, it is found that the lightest Higgs boson, whose main component is pseudoscalar, has a mass with about 36 GeV maximally and the sum of the masses of two lightest Higgs particles is around 93 GeV. So these particles are expected to be observed at LEP2 in the near future if the origin of the CP violation in the Higgs sector is reduced to the NMSSM with nontrivial phases of VEVs of two Higgs scalars (H_1^0, H_2^0) and a singlet scalar (N). The mass of charged Higgs is larger than 700 GeV, which is consistent with the present experimental lower limit for the charged Higgs mass.

Section 2 is devoted to the formulation of the NMSSM. In section 3, we discuss the framework of the experimental constraints and the spontaneous CP -violation scenario in the NMSSM. Section 4 gives parameters of the NMSSM and the masses of neutral and charged Higgs bosons by using the experimental constraints obtained in section 3. In section 5 we give summary and discussions.

2 Higgs Potential in the NMSSM and Higgs Masses

We study the spontaneous CP violation and the Higgs boson masses with radiative corrections of top, stop, bottom and sbottom fields in the NMSSM. Here the radiative effects of top superfield is essential and bottom superfield are significant especially in the case of large $\tan \beta$, so that the relevant terms in the superpotential is

$$W = h_t Q H_2 T^c + h_b Q H_1 B^c + \lambda N H_1 H_2 + \frac{k}{3} N^3, \quad (1)$$

where

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}. \quad (2)$$

with

$$H_1 H_2 = H_1^0 H_2^0 - H_1^- H_2^+.$$

The cubic term in N is introduced to avoid a Peccei-Quinn symmetry which would require the existence of a light pseudo-Goldstone boson when the

symmetry is broken by non-zero VEVs of Higgs fields. The superpotential W is scale invariant and Z_3 invariant which might interpret the weak scale baryogenesis[25].

Let us start with discussing the scalar potential for the fields H_1 , H_2 and N , which is given by $V = V_{\text{tree}} + V_{1\text{-loop}}$ as

$$V_{\text{tree}} = V_F + V_D + V_{\text{soft}} , \quad (3)$$

where

$$\begin{aligned} V_F &= |\lambda|^2[|H_1 H_2|^2 + |N|^2(|H_1|^2 + |H_2|^2)] + |k|^2|N|^4 + (\lambda k^* H_1 H_2 N^{*2} + \text{h.c.}), \\ V_D &= \frac{g_1^2 + g_2^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g_2^2}{2}(|H_1|^2|H_2|^2 - |H_1 H_2|^2) \\ V_{\text{soft}} &= m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 + m_N^2|N|^2 \\ &\quad + (\lambda A_\lambda H_1 H_2 N + \text{h.c.}) + \left(\frac{k A_k}{3} N^3 + \text{h.c.}\right) . \end{aligned} \quad (4)$$

Hereafter we discuss the possibility of spontaneous CP violation in the Higgs sector, so that we take the parameters $h_t, h_b, \lambda, k, A_\lambda, A_k$ to be all real[26]. It is well known that the radiative corrections are important to analyze Higgs spectra and also these corrections are essential to study spontaneous CP violation in the MSSM[9] and the NMSSM[13]. So the radiative corrections to the scalar potential at one-loop level are given by [27]

$$V_{1\text{-loop}} = \frac{1}{64\pi^2} \text{Str} M^4 \left(\ln \frac{M^2}{Q^2} \right), \quad (5)$$

where Str denotes the supertrace defined as

$$\text{Str} g(m^2) = \Sigma(-1)^{2J}(2J+1)g(m^2). \quad (6)$$

Here m^2 denotes the mass eigenvalues of a particle of spin J and in the case of squarks M^2 means 4×4 $\tilde{t}_L, \tilde{t}_R^c, \tilde{b}_L, \tilde{b}_R^c$ mass squared matrices, which can be written as

$$M_{\tilde{t}, \tilde{b}}^2 = \begin{pmatrix} \tilde{t}_L & \tilde{t}_R^c & \tilde{b}_L & \tilde{b}_R^c \\ m_{11}^2 & m_{12}^2 & m_{13}^2 & m_{14}^2 \\ m_{12}^{*2} & m_{22}^2 & m_{23}^2 & m_{24}^2 \\ m_{13}^{*2} & m_{23}^{*2} & m_{33}^2 & m_{34}^2 \\ m_{14}^{*2} & m_{24}^{*2} & m_{34}^{*2} & m_{44}^2 \end{pmatrix}, \quad (7)$$

where

$$\begin{aligned}
m_{11}^2 &= m_Q^2 + h_t^2 |H_2^0|^2 + h_b^2 |H_1^-|^2 - \frac{g_1^2}{12} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2) \\
&\quad + \frac{g_2^2}{4} (|H_1^0|^2 - |H_1^-|^2 - |H_2^0|^2 + |H_2^+|^2), \\
m_{12}^2 &= h_t (A_t H_2^{0*} + \lambda N H_1^0), \\
m_{13}^2 &= -h_t^2 H_2^{0*} H_2^+ - h_b^2 H_1^{-*} H_1^0 + \frac{g_2^2}{2} (H_2^+ H_2^{0*} + H_1^{-*} H_1^0), \\
m_{14}^2 &= -h_b (\lambda N H_2^+ - A_b H_1^{-*}), \\
m_{22}^2 &= m_T^2 + h_t^2 (|H_2^0|^2 + |H_2^+|^2) + \frac{g_1^2}{3} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2), \\
m_{23}^2 &= h_t (\lambda N^* H_1^{-*} - A_t H_2^+), \\
m_{24}^2 &= h_t h_b (H_2^0 H_1^{-*} + H_2^+ H_1^{0*}), \\
m_{33}^2 &= m_Q^2 + h_b^2 |H_1^0|^2 + h_t^2 |H_2^+|^2 - \frac{g_1^2}{12} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2), \\
&\quad + \frac{g_2^2}{4} (-|H_1^0|^2 + |H_1^-|^2 + |H_2^0|^2 - |H_2^+|^2), \\
m_{34}^2 &= -h_b (A_b H_1^{0*} + \lambda N H_2^0), \\
m_{44}^2 &= m_B^2 + h_b^2 (|H_1^0|^2 + |H_1^-|^2) - \frac{g_1^2}{6} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2).
\end{aligned}$$

The mass parameters m_Q, m_T, m_B are the soft supersymmetry breaking squark masses. Here the parameters A_t and A_b are the soft supersymmetry breaking ones corresponding to the first two terms of the superpotential Eq.(1);

$$V_{\text{soft}} = A_t h_t (\tilde{t}_L \tilde{t}_R^c H_2^0 - \tilde{b}_L \tilde{t}_R^c H_2^+) + A_b h_b (\tilde{t}_L \tilde{b}_R^c H_1^- - \tilde{b}_L \tilde{b}_R^c H_1^0) + \text{h.c.} \quad (8)$$

We also take A_t and A_b to be real in the present spontaneous CP violation scenario.

In order to realize spontaneous CP violation in the Higgs sector, it is necessary to have nonzero complex VEVs for H_1^0, H_2^0 and N . We define VEVs of Higgs fields as

$$\begin{aligned}
\langle H_1^0 \rangle &= v_1 e^{i\theta}, & \langle H_2^0 \rangle &= v_2, & \langle N \rangle &= x e^{i\xi/3}, \\
\langle H_1^- \rangle &= 0, & \langle H_2^+ \rangle &= 0,
\end{aligned} \quad (9)$$

where v_1, v_2 and x are all real and positive parameters.

In our scenario, Higgs sector can be parametrized in terms of 11 free parameters: the soft Higgs masses m_{H_1}, m_{H_2}, m_N , $\tan\beta$, x , phases of VEVs θ, ξ , the trilinear couplings in the superpotential λ and k and the soft scalar masses A_λ and A_k . The radiative corrections $V_{1\text{-loop}}$ due to top, stop, bottom and sbottom loops contain the soft top mass A_t and the soft bottom mass A_b and the squark mass parameters m_Q, m_B , and m_T . Then we have 16 parameters in total. By minimizing the Higgs potential with respect to the three VEVs and two phases, we can eliminate 5 parameters which are m_{H_1}, m_{H_2}, m_N, k and ξ by the equations

$$\frac{\partial}{\partial v_i} V = 0 \quad (i = 1, 2), \quad \frac{\partial}{\partial x} V = 0, \quad (10)$$

and

$$\frac{\partial}{\partial \theta} V = 0, \quad \frac{\partial}{\partial \xi} V = 0. \quad (11)$$

Then there remains 11 parameters which determine the masses and couplings of the five neutral and the charged Higgs bosons.

We can expand the neutral Higgs fields around their minimum points as

$$\begin{aligned} H_1^0 &= v_1 e^{i\theta} + \frac{1}{\sqrt{2}} e^{i\theta} (S_1 + i \sin\beta A), \\ H_2^0 &= v_2 + \frac{1}{\sqrt{2}} (S_2 + i \cos\beta A), \\ N &= x e^{i\xi/3} + \frac{1}{\sqrt{2}} e^{i\xi/3} (X + iY), \end{aligned} \quad (12)$$

where the five components are described as

$$\begin{aligned} S_1 &= \sqrt{2}(\cos\theta \text{Re}H_1^0 - \sin\theta \text{Im}H_1^0), \\ S_2 &= \sqrt{2} \text{Re}H_2, \\ A &= \sqrt{2}\{-\sin\beta(\sin\theta \text{Re}H_1^0 + \cos\theta \text{Im}H_1^0) + \cos\beta \text{Im}H_2\}, \\ X &= \sqrt{2}(\cos(\xi/3) \text{Re}N + \sin(\xi/3) \text{Im}N), \\ Y &= \sqrt{2}(-\sin(\xi/3) \text{Re}N + \cos(\xi/3) \text{Im}N). \end{aligned} \quad (13)$$

If the CP symmetry is conserved in the Higgs potential of the NMSSM, θ and ξ should set to be zero and the five neutral Higgs bosons are separated into three scalar bosons and two pseudoscalar bosons. The neutral Higgs mass

matrix in the spontaneous CP violation scenario induced by the one-loop effects is given as

$$M_H = \begin{pmatrix} \frac{1}{2} \frac{\partial^2 V}{\partial S_1^2} & \frac{\partial^2 V}{\partial S_1 \partial S_2} & \frac{\partial^2 V}{\partial S_1 \partial A} & \frac{\partial^2 V}{\partial S_1 \partial X} & \frac{\partial^2 V}{\partial S_1 \partial Y} \\ \frac{\partial^2 V}{\partial S_1 \partial S_2} & \frac{1}{2} \frac{\partial^2 V}{\partial S_2^2} & \frac{\partial^2 V}{\partial S_2 \partial A} & \frac{\partial^2 V}{\partial S_2 \partial X} & \frac{\partial^2 V}{\partial S_2 \partial Y} \\ \frac{\partial^2 V}{\partial S_1 \partial A} & \frac{\partial^2 V}{\partial S_2 \partial A} & \frac{1}{2} \frac{\partial^2 V}{\partial A^2} & \frac{\partial^2 V}{\partial A \partial X} & \frac{\partial^2 V}{\partial A \partial Y} \\ \frac{\partial^2 V}{\partial S_1 \partial X} & \frac{\partial^2 V}{\partial S_2 \partial X} & \frac{\partial^2 V}{\partial A \partial X} & \frac{1}{2} \frac{\partial^2 V}{\partial X^2} & \frac{\partial^2 V}{\partial X \partial Y} \\ \frac{\partial^2 V}{\partial S_1 \partial Y} & \frac{\partial^2 V}{\partial S_2 \partial Y} & \frac{\partial^2 V}{\partial A \partial Y} & \frac{\partial^2 V}{\partial X \partial Y} & \frac{1}{2} \frac{\partial^2 V}{\partial Y^2} \end{pmatrix}. \quad (14)$$

This matrix is diagonalized numerically and we obtain the physical Higgs fields h_i ($i = 1 \sim 5$). As for the mass of charged Higgs boson in the NMSSM with spontaneous CP violation, Babu and Barr presented the simple formula[13]

$$m_{H^\pm}^2 = M_W^2 + (3r - 1)\lambda^2 v^2, \quad (15)$$

where $r \equiv A_\lambda/A_k$. By using positivity condition of sub-determinants for squared mass matrix of neutral Higgs bosons and the local minimum condition for spontaneous CP -violation, they obtained the constraint

$$0 \leq (3r - 1)\lambda^2 \leq \frac{1}{2}\lambda_1(\sqrt{1 + \Delta} - 1), \quad (16)$$

where $\lambda_1 \equiv M_Z^2/v^2$ and Δ is a parameter given by the radiative effect at the limit of $m_{t_L}^2 = m_{t_R}^2$ with neglecting the contribution from bottom and sbottom loop. This constraint requires the upper limit of charged Higgs boson mass should be less than 110GeV. However, from the structure of squark mass matrix Eq.(7), off-diagonal elements, which do not exist in the analysis by Babu and Barr[13], receive the contribution of x . The large x raises the charged Higgs boson mass as shown in section 4 numerically.

3 Experimental constrains and the spontaneous CP -violation in the NMSSM

In the previous section we have obtained a 5×5 squared Higgs mass matrix M_H in Eq.(14). By diagonalizing this matrix the five eigenstates of Higgs

masses are derived and the five mass eigen states are defined as

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{pmatrix} = O \begin{pmatrix} S_1 \\ S_2 \\ A \\ X \\ Y \end{pmatrix}, \quad (17)$$

where the line of *l.h.s* is the order of masses, *i.e.* m_{h_i} is lighter than m_{h_j} for $i < j$. The orthogonal 5×5 matrix is defined as

$$(O)_{ij} \equiv a_{ij}. \quad (18)$$

The masses of these eigenstates should be positive. This condition means that the vacuum does not break QED in the charged Higgs sector. The components $M_{13,23,15,25,45}$ of mass squared matrix M_H are not zero when the CP symmetry is violated spontaneously. The magnitudes of these components are proportional to $\sin \eta$ or $\sin \xi$, where angle η is defined as

$$\eta \equiv \arg(H_1 H_2 N) = \theta + \frac{\xi}{3}. \quad (19)$$

In Ref.[13], Babu and Barr gave the analyses of the spontaneous CP violation in the NMSSM by using the following experimental constraints;

(i) the condition

$$m_{h_1} + m_{h_2} > M_Z \quad (20)$$

by the fact that Higgs bosons h_1 and h_2 have not been observed in the decay of Z [11] and

(ii) the lower mass limit is

$$m_{h_1} > (60\text{GeV})(\alpha_1 \cos \beta + \alpha_2 \sin \beta)^2, \quad (21)$$

where $h_1 \simeq \alpha_1 S_1 + \alpha_2 S_2$ by the experiment that the lightest boson h_1 has not been observed in the decay $Z \rightarrow h_1 + Z^* \rightarrow h_1 + l^+ l^-$ [28].

However, we should carefully analyze these conditions in the case of spontaneous CP -violation in the Higgs sector. First we estimate the coupling $g_{Zh_1 h_2}$ and discuss a possibility to be free from the experimental constraint

(i) in case of small $g_{Zh_1h_2}$ coupling even if the sum of two lightest Higgs boson masses is lighter than m_Z . The effective Hamiltonian for $Z \rightarrow h_1 + h_2$ is

$$H_{Zh_1h_2} = \frac{g_{Zh_1h_2}}{2 \cos \theta_W} Z_\mu (P_{h_1}^\mu - P_{h_2}^\mu), \quad (22)$$

and

$$g_{Zh_1h_2} \equiv g_2(\cos \beta(a_{12}a_{23} - a_{22}a_{13}) - \sin \beta(a_{11}a_{23} - a_{21}a_{13})). \quad (23)$$

In the case of $m_{h_1} + m_{h_2} < M_Z$, the decay $Z \rightarrow h_1 + h_2$ is physically possible and the decay rate is given as

$$\Gamma(Z \rightarrow h_1h_2) = \frac{M_Z}{16\pi} g_{Zh_1h_2}^2 \lambda^{\frac{3}{2}}(1, x_1, x_2), \quad (24)$$

where the familiar function $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and $x_i \equiv m_{h_i}^2/M_Z^2$. If $B(Z \rightarrow h_1h_2) < 10^{-7}$ the constraint (i) has no meanings, since we take the experimental limit for rare decays of Z to be 10^{-7} [11]. If the case $m_{h_1} + m_{h_2} > M_Z$ is realized, we should estimate the cross section for the process $e^-e^+ \rightarrow "Z" \rightarrow h_1h_2$. By using the coupling constant $g_{Zh_1h_2}$ and the Hamiltonian $H_{Zh_1h_2}$ we obtain

$$\sigma(e^-e^+ \rightarrow h_1h_2) = \frac{\alpha g_{Zh_1h_2}^2}{24 \sin^2 \theta_W \cos^2 \theta_W} \frac{1}{s} (|C_L|^2 + |C_R|^2) \frac{\lambda^{\frac{3}{2}}(1, y_1, y_2)}{(1 - y_Z)^2 + \frac{y_Z \Gamma_Z^2}{4s}}, \quad (25)$$

where $y_i = m_{h_i}^2/s$ and $y_Z = M_Z^2/s$.

As for the constraint (ii) we can give the similar argument to the case (i). The coupling constant g_{ZZh_1} and g_{ZZh_2} are given as

$$\begin{aligned} g_{ZZh_1} &\equiv \frac{g_2}{2 \cos \theta_W} M_Z \cos \beta (a_{11} + a_{12} \tan \beta), \\ g_{ZZh_2} &\equiv \frac{g_2}{2 \cos \theta_W} M_Z \cos \beta (a_{21} + a_{22} \tan \beta), \end{aligned} \quad (26)$$

respectively. Then if m_{h_1} and/or m_{h_2} are lighter than M_Z , the decay rate is

$$\begin{aligned} \Gamma(Z \rightarrow h_i l^+ l^-) &= \frac{1}{96\pi^3} \frac{g_{ZZh_i}^2 g_{Zl^+l^-}^2}{M_Z} (|C_L|^2 + |C_R|^2) \\ &\int_{\rho}^{\frac{1+\rho^2}{2}} \frac{1 + \rho^2 - 2x}{(\rho^2 - 2x)^2 + \Gamma_Z^2/(4M_Z^2)} (x^2 - \rho^2)^{1/2} dx, \end{aligned} \quad (27)$$

where $\rho = m_{h_i}/M_Z$, $x = E_{h_i}/M_Z$, $g_{Zl^+l^-} = 2e/\sin 2\theta_W$, $C_L = -\frac{1}{2} + \sin^2 \theta_W$ and $C_R = \sin^2 \theta_W$. This decay rate should be lower than the experimental upper bound Γ^{exp} , which is equivalent to $B(Z \rightarrow hl^+l^-) < 1.3 \times 10^{-7}$ at $m_h = 60\text{GeV}$ in the SM[11]:

$$\Gamma(Z \rightarrow h_i l^+ l^-) < \Gamma^{\text{exp}}. \quad (28)$$

For h_1 and h_2 , we use this constraint instead of Eq.(21) in our spontaneous CP violation scenario. It is noted that the constraint of Eq.(21) is weaker than ours because it does not take into account the phase space integral. It is found that our constraint almost rules out solutions given by Babu and Barr[13]. These constraints for the masses m_{h_i} are discussed numerically in the next section.

Summarizing the above arguments, we use the following experimental constraints in the next section;

- A** if the sum of lightest Higgs bosons m_{h_1} and m_{h_2} is lighter than m_Z , the branching ratio $B(Z \rightarrow h_1 h_2)$ should be less than 10^{-7} or the sum of m_{h_1} and m_{h_2} should be larger than m_Z and
- B** for h_1 and h_2 , both of $B(Z \rightarrow h_1 l^+ l^-)$ and $B(Z \rightarrow h_2 l^+ l^-)$ should be smaller than 1.3×10^{-7} . Hereafter we call the former constraint as constraint **B1** and the latter as constraint **B2**.

4 Numerical results on the spontaneous CP Violation

In this section we analyze about the parameters $\tan \beta, \lambda, \eta, A_\lambda, A_t$ etc. in the spontaneous CP-violation scenario numerically. Assuming the perturbation remains valid up to the unification scale the couplings λ and k are restricted by their fixed points as pointed out by Ellis et al. in Ref.[20] such as

$$|\lambda| \leq 0.87, \quad |k| \leq 0.63. \quad (29)$$

We use these theoretical constraints to restrict the parameters in the followings because the spontaneous CP violation gives no change for the renormalization group equation of the real parameters λ and k [15].

As mentioned in section 2, the minimization conditions Eqs.(10,11) of Higgs potential determine the soft Higgs masses m_{H_1}, m_{H_2}, m_N , the phase ξ and N^3 coupling constant k . The parameters ξ and k are given by

$$D \sin \eta \cos \xi - (D \cos \eta + F) \sin \xi = 0 \quad (30)$$

and

$$k = \frac{1}{\lambda \sin(\eta - \xi)} \left(\frac{1}{2v_1 v_2 x^2} \frac{\partial V_{1\text{-loop}}}{\partial \eta} - E \sin \eta \right), \quad (31)$$

respectively, where the definition of D, E and F are followed by Ref.[13] as

$$D = \lambda k, \quad E = \frac{\lambda A_\lambda}{x}, \quad F = \frac{k A_k x}{3v_1 v_2}. \quad (32)$$

We use quark masses and the coupling constants as

$$\begin{aligned} m_t &= 174\text{GeV}, & m_b &= 4.2\text{GeV}, & g_1 &= 0.357, \\ g_2 &= 0.625, & h_t &= 174.0/v_2, & h_b &= 4.2/v_1. \end{aligned} \quad (33)$$

The parameters A_t and m_Q are given in order to satisfy the necessary condition not to break color symmetry in the squark sector[15][20]. The remaining parameters A_b, m_T, m_B are fixed by the arguments of fixed point analyses with the assumption of GUT scale universality[15] as

$$A_b = 1.1A_t, \quad m_T = 0.95m_Q, \quad m_B = 0.98m_Q, \quad (34)$$

where the renormalization point is taken as $Q = 3.0\text{TeV}$. Under the above mentioned experimental constraints A and B we search the relevant parameter region. The allowed parameter ranges are rather narrow. In order to compare our result with the one given by Babu-Barr [13], we show the following typical set of parameters, which satisfy constraints A and B, are

$$\begin{aligned} \eta &= 1.275, & \tan \beta &= 1.0, & \lambda &= 0.16, & A_k &= 2.9v, & A_\lambda/A_k &= 0.8 \\ x &= 3.8v, & A_T &= 1\text{TeV}, & m_Q &= 3\text{TeV}, \end{aligned} \quad (35)$$

where the parameter k takes the value -0.612 . In this case the Higgs masses are obtained as

$$\begin{aligned} m_{h_1} &= 35.8\text{GeV}, & m_{h_2} &= 57.0\text{GeV}, & m_{h_3} &= 177\text{GeV}, \\ m_{h_4} &= 671\text{GeV}, & m_{h_5} &= 785\text{GeV}. \end{aligned} \quad (36)$$

The constraint B is much severer than the constraint (ii) which Babu-Barr used [13]. The allowed regions obtained by Babu-Barr are almost excluded if we use constraint B. For example, if η is shifted with only ± 0.01 , the solution does not satisfy the constraint B. Then, one should shift λ with ± 0.05 in order to get allowed solution. Thus, the allowed parameter set is very restrictive in contrast with the result given by Babu-Barr[13]. We will show the results of other parameter dependence later.

For the case of parameters in Eq.(35), the components of each Higgs boson are given as

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{pmatrix} = \begin{pmatrix} 0.255 & 0.058 & 0.965 & 0.012 & 0.028 \\ 0.950 & 0.167 & -0.262 & 0.002 & 0.037 \\ -0.177 & 0.984 & -0.013 & -0.003 & 0.031 \\ 0.005 & 0.005 & 0.025 & 0.999 & -0.031 \\ -0.037 & -0.038 & -0.016 & 0.032 & 0.998 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ A \\ X \\ Y \end{pmatrix}, \quad (37)$$

where (1,1), (1,2), (2,1) and (2,2) components are same signs. Since the two terms in the *r.h.s.* of Eq.(26) are additive, the coupling constants g_{ZZh_1} and g_{ZZh_2} are not remarkably reduced. This situation is different from that in the MSSM, where couplings are somewhat reduced. Thus, the constraint B for the NMSSM is severer than the one for the MSSM. It is remarked that the lightest Higgs state mainly consists of pseudoscalar component A as shown in Eq.(37).

In figure 1 we give the cross section for $e^-e^+ \rightarrow h_1h_2$ from the threshold to $\sqrt{s}/2=200\text{GeV}$ and at the energy of LEP1.5 the production cross section is about 0.8pb in the case of Eq.(37).

Fig.1

For the charged Higgs boson, its squared mass is given by taking the coefficient of the twice derivative of $V = V_F + V_D + V_{\text{soft}} + V_{1\text{-loop}}$ by H^- and H^+ using Eqs.(3,6,7), where the physical charged Higgs is defined as

$$h^+ \equiv \cos \beta H_2^+ - \sin \beta H_1^-. \quad (38)$$

The numerical results for the parameters in Eq.(35) are

$$m_{h^\pm} = 721\text{GeV}, \quad (39)$$

which depends crucially on the squark mass m_Q . We show the m_Q dependence of the charged Higgs mass in figure 2, in which other parameters are fixed as in Eq.(35). The upper bound of m_Q is given by $|k| < 0.63$ in Eq.(29) and lower bound by constraint B. Thus, m_Q should be larger than 3TeV. The predicted charged Higgs mass is too large to detect this boson at LEP2 and this mass becomes free from the constraints of $b \rightarrow s\gamma$ experiment[22].

Fig.2

It is noticed that the sum of two masses is almost constant around 93GeV even if the other parameter set which fulfills the constraints A and B is taken. Therefore, these two Higgs bosons will be observed at LEP2 experiment in the near future. In the present study we obtain rather lighter Higgs masses compared to the case without spontaneous CP violation in the NMSSM[20][21]. This circumstances are understood by the Georgi-Pais theorem for the radiative symmetry breaking phenomena[10]. It is also noted that the two lower Higgs masses are almost independent of the parameter x , where other parameters are fixed as in Eq.(35).

It may be useful to comment on the value of $\tan\beta$. There is no solution for the spontaneous CP violation in the range of $\tan\beta > 1$ through the numerical analyses. In case of the MSSM, the arguments on electroweak symmetry breaking and the top Yukawa coupling lead to the allowed ranges for $\tan\beta$ as $1.0 \leq \tan\beta \leq 1.4$ [29] although the large top quark mass does not prefer $\tan\beta \simeq 1$ in the RGE analyses of the Yukawa couplings. If $\tan\beta = 1$ is completely ruled out in SUSY, our scenario could not be realized for the CP violation. Thus, the value of $\tan\beta$ is the critical quantity for our scheme.

So we investigate the available x region being consistent with the current experimental constraints A and B, where the parameter A_λ and A_t are freely adjusted with the fixed value of $\tan\beta = 1$. It is found that the solutions exist for $x \geq 2v$ and we show the typical solution for $x = 20v$ as an example of large x case for the comparison of the relatively small x case Eq.(35).

$$\begin{aligned}
\eta &= 1.3, & \tan\beta &= 1.0, & \lambda &= 0.16, & A_k &= 16v, & A_\lambda &= 12.5v \\
x &= 20v, & A_T &= 1\text{TeV}, & m_Q &= 3\text{TeV} .
\end{aligned}
\tag{40}$$

In this case the Higgs masses are obtained as

$$\begin{aligned}
m_{h_1} &= 35.9\text{GeV}, & m_{h_2} &= 57.2\text{GeV}, & m_{h_3} &= 177\text{GeV}, \\
m_{h_4} &= 3584\text{GeV}, & m_{h_5} &= 4261\text{GeV}.
\end{aligned}
\tag{41}$$

The charged Higgs masses is 721GeV, which is not changed as far as $m_Q = 3\text{TeV}$ is fixed.

The allowed region of $A_t - x$ plane is shown in figure 4, in which the inside region of the triangle is allowed. It is emphasized that $A_t = 0$ is not allowed. In other words, the full radiative correction at one loop level, which Babu-Barr did not take into consideration, is significant to study spontaneous CP violation in the NMSSM.

Fig.3

In Ref.[13], they analyzed the spontaneous CP violation and obtained the region of λ versus $\cos\eta$. The available region of λ and $\cos\eta$ is not so similar to our results as mentioned above. This shows that the constraint B is also important as well as the full radiative correction at one loop level.

Without spontaneous CP violation the Higgs masses and other parameters in the NMSSM are widely analyzed by many authors[21]. It is well known that the NMSSM with radiative correction yields the heavier mass for the lightest CP even scalar to be around 130GeV independently on the top quark mass as shown by Elliot et al. in Ref.[21].

5 Summary and Discussion

We have studied the spontaneous CP violation in the NMSSM by including the full one-loop radiative effects into the Higgs potential. The parameter

region being compatible with the current lower bounds for Higgs masses has been analyzed.

The experimental upper bound $B(Z \rightarrow hl^+l^-)$ gives the very severe constraints on the solution of spontaneous CP violation. The available region of parameters are very narrow. We have obtained the large spontaneous CP violation as $\eta \simeq 1.3$. The solution only exists around $\tan\beta \simeq 1.0$ and in the vicinity of 0.16 for the coupling λ .

The upper limit of the lightest neutral Higgs h_1 is 36GeV for all available parameter regions. Also the total mass of the lightest h_1 and the second lightest Higgs boson h_2 is almost constant and around 93GeV. The charged Higgs mass is around 700GeV, which depends on m_Q . The predicted charged Higgs mass is too large to detect this boson at LEP2 and this mass is free from the constraints of $b \rightarrow s\gamma$ experiment.

However, if the experimental upper bound $B(Z \rightarrow hl^+l^-)$ will be improved in factor 1.5, one has no more solution of spontaneous CP violation in the NMSSM.

Since CP violation in the Higgs sector does not occur in the MSSM without a gauge singlet Higgs field N , CP violation is an important signal of the existence of the gauge singlet Higgs field. The lightest Higgs mass in the NMSSM without spontaneous CP violation could be larger than 130GeV and it is expected that the LEP2 experiment will give the solution on the possibility of spontaneous CP violation in the Higgs sector. In the present case of the Higgs sector, the analyses of the electron and the neutron EDM and the production and the decay of Higgs state mixed with scalar and pseudoscalar components will be given in the forthcoming paper.

Acknowledgments

This research is supported by the Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, Japan(No.06640386, No.07640413).

References

- [1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652(1973).
- [2] For a text of Higgs physics see J.F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson,
"Higgs Hunter's Guide", Addison-Wesley, Reading, MA(1990).
- [3] T. D. Lee, Phys. Rev. **D8**, 1226(1973).
- [4] G.C. Branco and M.N. Rebelo, Phys. Lett. **160B**, 117(1985).
- [5] S. Weinberg, Phys. Rev. **D42**, 860(1990).
- [6] U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. **260B**, 447(1991).
- [7] T. Hayashi, Y. Koide, M. Matsuda, and M. Tanimoto, Prog. Theor. Phys. **91**(1994),915.
- [8] Y. Okada, M. Yamaguchi, and T. Yanagida, Prog. Theor. Phys. **85**(1991),1;Phys. Lett. **B262**(1991),54.
J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett.**B257**(1991),83.
H. Haber and R. Hempfling, Phys. Rev. Lett.**66**(1991),1815.
R. Barbieri, M. Frigeni, and F. Caravaglios, Phys. Lett. **B258**(1991),167.
A. Yamada, Phys. Lett.**B263**(1991),233.
A. Brignole, J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. **B271**(1991), 123
- [9] N. Maekawa, Phys. Lett. **B282** (1992), 387.
A. Pomarol, Phys. Lett. **B287** (1992), 331.
- [10] H. Georgi and A. Pais, Phys. Rev. **D10** (1974), 1246.
- [11] Particle Data Group, M. Aguilar-Benitez et al., Phys. Rev. **D50**(1994), 1173
- [12] M. Matsuda and M. Tanimoto, Phys. Rev. **D52**(1995), 3100 .
- [13] K.S. Babu and S.M. Barr, Phys. Rev. **D49** (1994), 2156.

- [14] P. Fayet, Nucl. Phys. **B90**(1975), 104.
 R.K. Kaul and P. Majumdar, Nucl. Phys. **B199**(1982), 36.
 R. Barbieri, S. Ferrara and C.A. Sávoy, Phys. Lett. **119B**(1982), 343.
 H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. **120B**(1983), 346.
 J.M. Frère, D.R.T. Jones and S. Raby, Nucl. Phys. **B222**(1983), 11.
- [15] J.P. Derendinger and C.A. Savoy, Nucl. Phys. **B237**(1984), 307.
- [16] H. P. Nilles, Phys. Rep. **C110** (1984), 1.
 P. Nath, R. Arnowitt and A. H. Chamseddine, "Applied $N=1$ supergravity,
 World Scientific, Singapore (1984), p48.
- [17] E. Cohen, J. Ellis, K. Enqvist and D. V. Nanopoulos, Phys. Lett. **B165**(1985),76.
 J. P. Derendinger, L. E. Ibanez and H. P. Nilles, Nucl. Phys. **B267**(1986), 365.
 P. Bonetruy, S. Dawson, I. Hinchliffe and M. Sher, Nucl. Phys. **B273**(1986), 501
- [18] F. Zwirner, Int. J. Mod. Phys. **A3**(1988) 49.
 N. Haba, C. Hattori, M. Matsuda, T. Matsuoaka and D. Mochinaga, Phys. Lett. **B337**(1994), 63; Prog. Theor. Phys. **94** (1995),233; *ibid* **95** (1996), 191.
- [19] J. Ellis, J. S. Hagelin, S. Kelly and D. V. Nanopoulos, Nucl. Phys. **B311**(1988),1.
- [20] J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. **D39**(1989),844.
- [21] P. N. Pandita, Z. Phys. **C59**(1993), 575.
 F. Franke, H. Fraas and A. Bartl, Phys.Lett. **B336**(1994),415.
 T. Elliott, S. F. King and P. L. White, Phys. Rev. **D49**(1994),2435; Phys. Lett. **B314**(1993),56;*ibid* **B351**(1995),213.
 S. F. King , P. L. White, Phys. Rev. **D52** (1995), 4183.
- [22] CLEO Collaboration (M.S. Alam et al.), Phys. Rev. Lett.**74**(1995),2885.

- [23] T. Goto and Y. Okada, Prog. Theor. Phys.**94**(1995),407.
- [24] T. Hayashi, M. Matsuda and M. Tanimoto, Prog. Theor. Phys.**89**(1993),1047.
- [25] S. A. Abel and P. L. White, Phys. Rev. **D52**(1995), 4371.
- [26] The possibility of explicit CP violation in the Higgs sector in the NMSSM is discussed in Ref.[12].
- [27] S. Coleman and E. Weinberg, Phys. Rev. **D7** (1973), 1888;
S. Weinberg, *ibid.* **7**, (1973) 2887.
- [28] ALEPH Collaboration, D. Buskulic et al., Phys. Lett. **B313**(1993), 312.
- [29] V. A. Bednyakov, W. de Boer and S. G. Kovalenko, Report No. hep-ph/9406419, June 1994(to be published).

Figure Captions

Fig.1 The cross section of $e^-e^+ \rightarrow h_1 + h_2$ versus $\sqrt{s}/2$ in the case of the solution given in Eq.(35).

Fig.2 The m_Q dependence of the charged Higgs mass.

Fig.3 The allowed region on $A_t - x$ plane constrained by a constraint **B1** (dashed line), a constraint **B2**(dotted line), a constraint **A** (dash-dotted line) and a constraint $|k| < 0.63$ (solid line), where constraints are explained in section 3 of the text.

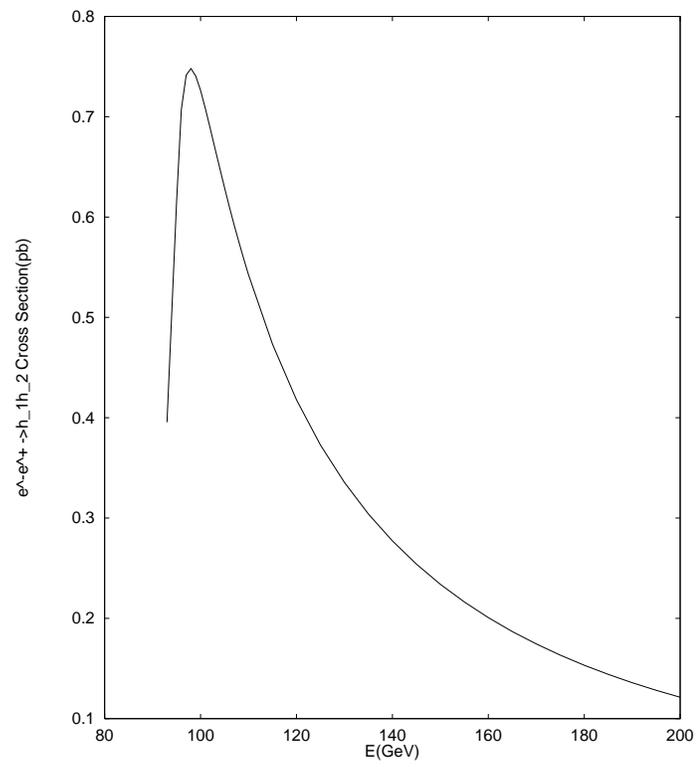


Fig.1

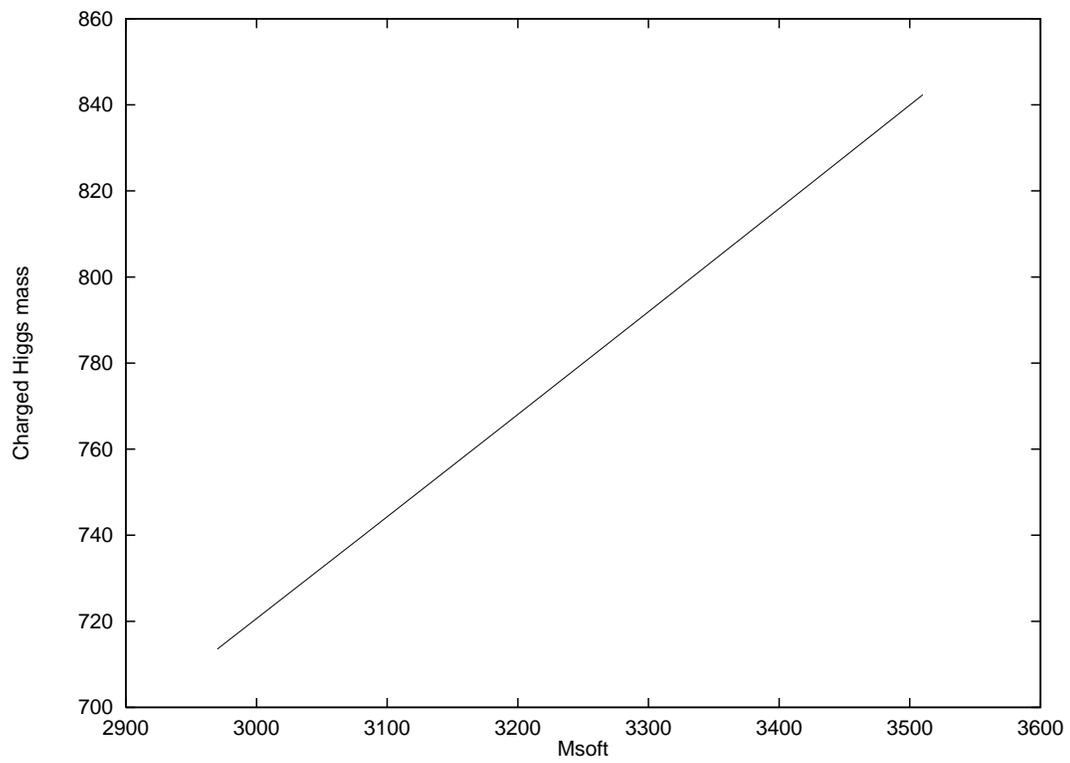


Fig.2

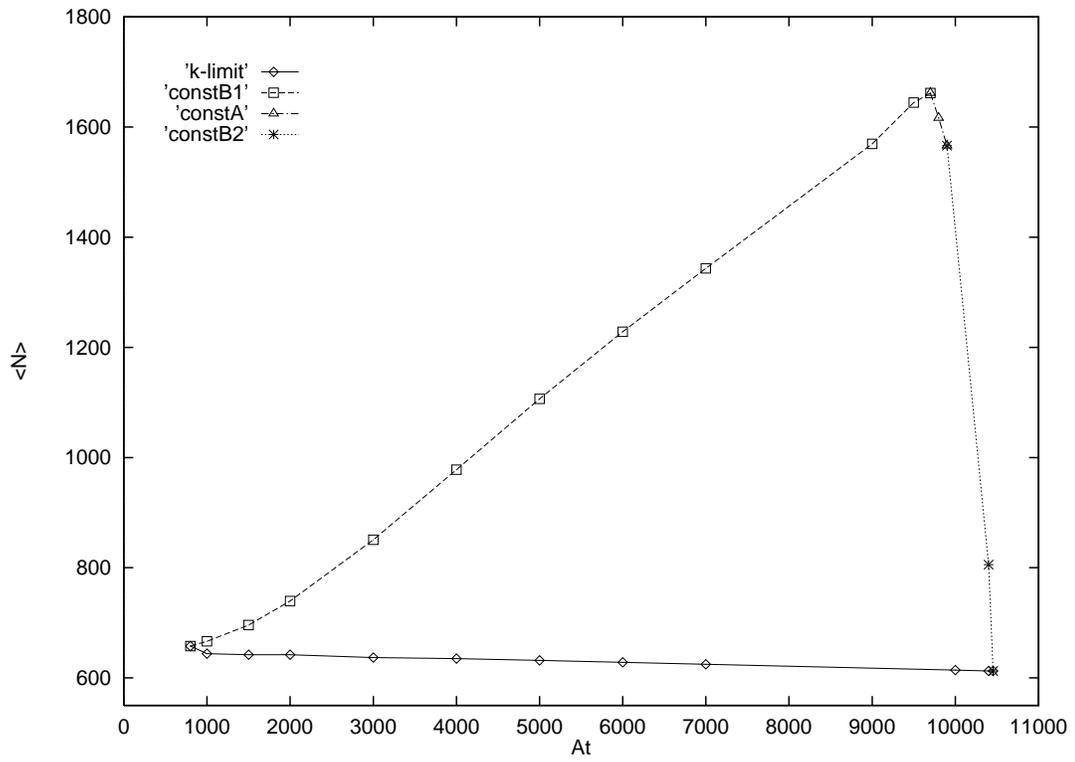


Fig.3

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9512421v3>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9512421v3>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9512421v3>