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# The Weak-Scale Hierarchy and Discrete Symmetries

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## Abstract

In the underlying Planck scale theory we introduce a certain type of discrete symmetry, which potentially brings the stability of the weak-scale hierarchy under control. Under the discrete symmetry the  $\mu$ -problem and the tadpole problem can be solved simultaneously without relying on some fine-tuning of parameters. Instead, it is required that doublet Higgs and color-triplet Higgs fields reside in different irreducible representations of the gauge symmetry group at the Planck scale and that they have distinct charges of the discrete symmetry group.

Recently, it is greatly expected that many characteristic features of low-energy effective theory are attributable to various types of symmetry in the underlying Planck scale theory, such as in superstring theory. It is plausible that the gauge symmetry  $G$  at the Planck scale is larger than the standard gauge group  $G_{st} = SU(3)_C \times SU(2)_L \times U(1)_Y$ . Since the larger  $G$  should be broken to  $G_{st}$ , some  $G_{st}$ -neutral fields are needed to be contained in the theory and to develop non-zero vacuum expectation values (VEVs) at some intermediate energy scales. And furthermore, it is likely that there exist certain discrete symmetries at the Planck scale. As suggested from Gepner model [1], such symmetries may have their origin in symmetrical structure of compactified space in superstring theory. The discrete symmetries put some restrictions on interactions including various couplings related to  $G_{st}$ -neutral fields. Then restricted couplings of  $G_{st}$ -neutral fields to the other fields reflect on the low-energy effective theory. In addition, the magnitude of VEVs of  $G_{st}$ -neutral fields would be governed by the discrete symmetry and small ratios of the VEVs to the Planck scale would yield the hierarchical structure to the effective theory. Consequently, the discrete symmetries and  $G_{st}$ -neutral fields would constitute vital ingredients of determining hierarchical structure of the effective theory.

In constructing realistic unified models we need to treat with a large hierarchy between two mass scales, i.e. the unification scale and the electroweak scale [2]. In general, such models are confronted with the so-called hierarchy problem. Namely, the weak-scale hierarchy is destabilized by quadratically divergent radiative corrections. Supersymmetry (SUSY) is an attractive idea to cure partially this problem and renders the hierarchy technically natural [3]. However, the lightness of Higgs doublets at the tree level is not assured by SUSY and some fine-tuning of parameters is needed at the tree level [4]. For example, in the minimal SUSY  $SU(5)$  GUT there appear  $\mathbf{5}$  and  $\mathbf{5}^*$  Higgs superfields, in which Higgs doublets  $(H_u, H_d)$  and Higgs triplets  $(g, g^c)$

are contained. From proton stability Higgs triplets should be superheavy. While Higgs doublets should not be superheavy since they constitute an important ingredient of the low-energy model. Their mass terms in the superpotential are assumed to be

$$W \sim \mu H_u H_d + M_g g g^c \quad (1)$$

with  $\mu = O(10^2 \text{GeV})$  and  $M_g = O(M_{\text{GUT}})$ . Why is  $\mu$  the electroweak scale but not the unification scale? This is the so-called  $\mu$ -problem. Although SUSY protects this mass hierarchy  $M_g \gg \mu$  against radiative corrections, the mass hierarchy at tree level have to be fine-tuned. Many other GUT models also suffer from the  $\mu$ -problem. When Higgs doublets and Higgs triplets belong to the same irreducible representations of  $G$ , such as to  $\mathbf{5}$  and  $\mathbf{5}^*$  of  $SU(5)$ , coupling constants of Higgs doublets and of Higgs triplets to the singlet or adjoint Higgs are of the same order. In order to get triplet-doublet mass splitting without fine-tuning, we are enforced to introduce additional Higgs fields with the larger representations of  $G$ . However, it seems that these enlargements of the Higgs sector are rather complicated and bring about another problem to the models [5]. Therefore, it is likely that Higgs doublets and Higgs triplets reside in different irreducible representations of the gauge group  $G$  at the Planck scale.

In anticipation of explaining  $\mu = O(10^2 \text{GeV})$ , several authors introduced a  $G_{st}$ -neutral field  $N$  [6], provided that Higgs triplets have a large mass  $M_g$  whereas Higgs doublets remain massless at the unification scale. In this scenario there exist trilinear couplings with  $N$ . The superpotential is given by

$$W \sim M_g g g^c + f_H N H_u H_d + f_g N g g^c . \quad (2)$$

Unless we have some kinds of selection rule on trilinear couplings, the coupling constants  $f_H$  and  $f_g$  are to be  $O(1)$ . Suppose  $N$  develops a nonzero VEV with  $O(10^2 \text{GeV})$

via some mechanism, the  $\mu$ -term is induced as

$$\mu = f_H \langle N \rangle = O(10^2 \text{GeV}). \quad (3)$$

Even if this is the case, however, we encounter a new hierarchical problem. A trilinear coupling of the singlet  $N$  to superheavy Higgs triplets  $g, g^c$  brings about large mass correction to Higgs doublet scalar fields through tadpole diagrams as shown in Fig.1.

**Fig. 1**

The contribution of tadpole diagrams to Higgs scalar mass is given by

$$\delta m_{H_u, H_d}^2 \sim \frac{M_g f_g f_H m_{3/2}^3}{m_N^2}, \quad (4)$$

where  $m_N$  represents the scalar mass of  $N$ . Since the soft SUSY breaking terms give the scalar mass,  $m_N$  becomes of the order of  $m_{3/2} = O(1\text{TeV})$ . This mass correction is extremely large compared to  $O(m_{3/2}^2)$ . Thus the coupling of  $N$  to  $g, g^c$  destabilizes the mass hierarchy  $M_g \gg \mu$ . This is the so-called tadpole problem or light singlet problem [7].

In this paper we propose a new model with a certain type of discrete symmetry. The discrete symmetry implies a stringent selection rule on renormalizable and non-renormalizable interactions given by the superpotential. In the model a mirror pair of  $G_{st}$ -neutral fields  $N$  and  $\bar{N}$  is contained and develops a very large VEV  $\langle N \rangle = \langle \bar{N} \rangle$ . Without relying on some fine-tuning among parameters we obtain the relations

$$\mu = f_H \langle N \rangle, \quad f_H < O\left(\frac{m_{3/2}}{\langle N \rangle}\right), \quad f_g = O(1) \quad (5)$$

for effective couplings, respectively, with  $M_g = O(\langle N \rangle) \gg m_{3/2}$  and  $m_N^2 = O(m_{3/2}^2)$ . The smallness of the coupling  $f_H$  is explained naturally from the discrete symmetry.

It follows that

$$\mu < O(m_{3/2}), \quad \delta m_{H_u, H_d}^2 = O(\mu m_{3/2}). \quad (6)$$

In this model the  $\mu$ -problem and the tadpole problem are closely linked together and solved simultaneously.

In the model proposed here we are based on the following scheme of superstring or supergravity models. The gauge symmetry group  $G$  at the Planck scale  $\Lambda_P$  is rank-five or rank-six, such as in  $E_6$ -inspired models. In matter superfields there appear doublet Higgs superfields  $H_u, H_d$  and color-triplet Higgs superfields  $g, g^c$  which reside in distinct irreducible representations of  $G$ . In addition to these chiral superfields, we have a mirror pair of  $G_{st}$ -neutral but  $G$ -charged chiral superfields  $N$  and  $\bar{N}$ . Existence of mirror fields is likely in superstring models. It is supposed that as far as gauge invariance is concerned, the couplings  $NH_uH_d$  and  $Ngg^c$  are allowed whereas the couplings  $\bar{N}H_uH_d$  and  $\bar{N}gg^c$  are forbidden.

Let us introduce certain discrete symmetries at the Planck scale, which may be a reflection of the geometrical structure of the compactified space. In fact, peculiar discrete symmetries come into Gepner model in which the compactified space is constructed algebraically by a tensor product of  $N = 2$  superconformal field theory [1]. Concretely, the discrete symmetry  $Z_{k+2}$  or  $Z_{k+2} \times Z_2$  is derived from  $N = 2$  superconformal field theory with the level  $k$  in which each matter superfield has a distinct charge of the discrete symmetries. The discrete symmetries put a stringent selection rule on allowed couplings in the superpotential. In the present model it is assumed that allowed couplings are given by

$$W = \frac{\lambda_H}{\Lambda_P^{2p}} (N\bar{N})^p NH_uH_d + \lambda_g Ngg^c + \frac{\lambda_N}{\Lambda_P^{2l-1}} (N\bar{N})^{l+1} + \dots, \quad (7)$$

where  $1 \leq p, l$  and the coefficients  $\lambda_H, \lambda_g, \lambda_N$  are  $O(1)$ . As we will see later, the exponents  $p$  and  $l$  are determined according as the discrete charges of the matter

superfields. Our assumption contains that there appears a trilinear coupling only for colored Higgs fields because of special values of the discrete charge of the products  $(H_u H_d)$  and  $(gg^c)$ .

Incorporating the soft SUSY breaking terms, we can get the scalar potential  $V$ . The scale of SUSY breaking ( $m_{3/2}$ ) is supposed to be  $O(1\text{TeV})$ . The running scalar masses squared  $m_N^2$  and  $m_{\bar{N}}^2$  for  $N$  and  $\bar{N}$  are  $O(m_{3/2}^2)$ . Since  $N$  couples to colored Higgs with a sizable trilinear coupling constant,  $m_N^2$  possibly becomes negative even at large energy scale [8]. When  $m_N^2 + m_{\bar{N}}^2 < 0$ ,  $N$  and  $\bar{N}$  develop nonzero VEVs. By minimizing  $V$ , we obtain the VEVs [9] [10]

$$\langle N \rangle = \langle \bar{N} \rangle \equiv \Lambda \sim \Lambda_P \left( \frac{m_{3/2}}{\Lambda_P} \right)^{1/2l}, \quad (8)$$

which is sufficiently large compared to  $m_{3/2}$ . For instance, we have  $\Lambda \gtrsim 10^{16}\text{GeV}$  for  $l \geq 3$ . The magnitude of the scale  $\Lambda$  is controlled by the discrete charges of  $N$  and  $\bar{N}$ . Although spontaneous breaking of the gauge symmetry occurs at the scale  $\Lambda$ , the  $D$ -flatness condition is satisfied and then SUSY is preserved at this scale. In the symmetry breaking a combination  $(N - \bar{N})/\sqrt{2}$  is absorbed by a vector superfield due to the Higgs mechanism. The remaining component  $(N + \bar{N} - 2\Lambda)/\sqrt{2}$  ( $\equiv N'$ ) has a mass of order  $O(m_{3/2})$  irrespective of  $l$ . For the sake of convenience we introduce the notation  $x$  defined by

$$x = \frac{\Lambda}{\Lambda_P} \sim \left( \frac{m_{3/2}}{\Lambda_P} \right)^{1/2l}. \quad (9)$$

This small ratio  $x$  becomes an efficient parameter in describing the hierarchical structure of the effective theory.

Now we proceed to study the low-energy effective superpotential  $W^{eff}$  below the scale  $\Lambda$ . From Eqs. (7) to (9) the bilinear terms in  $W^{eff}$  becomes

$$W_2^{eff} = \mu H_u H_d + M_g gg^c + M_N N'^2 \quad (10)$$

with

$$\mu \simeq \lambda_H x^{2p+1} \Lambda_P, \quad (11)$$

$$M_g = \lambda_g \Lambda = \lambda_g x \Lambda_P, \quad (12)$$

$$M_N \simeq \lambda_N x^{2l} \Lambda_P \simeq m_{3/2}. \quad (13)$$

Colored Higgs fields get a mass of  $O(\langle N \rangle)$ , while doublet Higgs mass  $\mu$  is controlled by the exponent  $p$ . Explicitly,  $\mu$  is given by

$$\mu \simeq x^{2(p-l)+1} m_{3/2}. \quad (14)$$

Therefore, when  $p \geq l$ , the  $\mu$ -problem is solved. In what follows we take the condition

$$p \geq l \quad (15)$$

and then  $\mu \lesssim x m_{3/2} < O(1\text{TeV})$ . For example, we obtain  $\mu = O(10^2\text{GeV})$  for  $p = l \sim 8$ .

To address ourselves to the tadpole problem, we study the trilinear terms in  $W^{eff}$  which are of the form

$$W_3^{eff} = f_H N' H_u H_d + f_g N' g g^c + f_N N'^3, \quad (16)$$

where

$$f_H \simeq \lambda_H x^{2p}, \quad f_g = \lambda_g / \sqrt{2}, \quad f_N \simeq \lambda_N x^{2l-1}. \quad (17)$$

As a consequence of the discrete symmetry it follows that we have

$$f_g = O(1) \quad (18)$$

for  $N' g g^c$  coupling, whereas

$$f_H = \frac{\mu}{\Lambda} \ll 1 \quad (19)$$

for  $N'H_uH_d$  coupling. The tadpole contribution to the Higgs mass becomes

$$\delta m_{H_u, H_d}^2 \simeq \frac{M_g f_g f_H m_{3/2}^3}{m_N^2} \simeq M_g f_H m_{3/2} \simeq \mu m_{3/2}. \quad (20)$$

This implies that the tadpole problem is solved simultaneously together with the  $\mu$ -problem under the condition (15). This is due to the fact that both the  $\mu$ -term and the trilinear coupling  $N'H_uH_d$  are induced from the nonrenormalizable interaction  $(N\bar{N})^p NH_uH_d$  in the underlying theory.

For illustration we take up  $Z_\alpha$  as a simple example of the discrete symmetry, where  $\alpha$  is an integer larger than one. As mentioned above, this type of discrete symmetry possibly comes into Calabi-Yau string models. In Table I we tabulate  $Z_\alpha$ -charges of matter superfields, where  $b$  and  $c$  represent  $Z_\alpha$ -charges of the products  $(H_uH_d)$  and  $(gg^c)$ , respectively. Generally, as is the case with Gepner model, Grassmann number  $\theta$  also has a nonzero charge denoted as  $-d$  in Table I. Each charge is taken as  $0 \leq a, \bar{a}, b, c, d < \alpha$ . Since the superpotential (7) is assumed to be a consequence of the  $Z_\alpha$  symmetry, we have the relations

$$\left. \begin{aligned} p(a + \bar{a}) + a + b + 2d &\equiv 0 \\ a + c + 2d &\equiv 0 \\ (l + 1)(a + \bar{a}) + 2d &\equiv 0 \end{aligned} \right\} \text{mod } \alpha, \quad (21)$$

where  $1 \leq p, l < \alpha$ . When  $a, \bar{a}, b, d$  are given,  $c$  and the exponents  $p$  and  $l$  are determined from these equations. To get a nontrivial solution, the conditions  $a + \bar{a}, b - c \not\equiv 0 \pmod{\alpha}$  should be satisfied. The above relations lead to

$$(p - l)(a + \bar{a}) + b - \bar{a} \equiv 0 \pmod{\alpha}. \quad (22)$$

Thus, if  $b \equiv \bar{a}$  and if  $a + \bar{a}$  is prime to  $\alpha$ , we obtain

$$p = l. \quad (23)$$

This case is in accord with the condition (15). More concretely, when  $a = \bar{a} = b = d = 1$  and  $\alpha$  is odd, we get  $p = l = c + 1 = \alpha - 2$ .

<b>Table I</b>
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As for the generation structure of the  $G_{st}$ -neutral fields, so far it is postulated that we have only a pair of  $N$  and  $\bar{N}$ . Generally, however, the multiplicities of  $N$  and of the mirror superfield  $\bar{N}$  do not coincide with each other but rather in superstring models the difference of these multiplicities corresponds to the generation number. Taking this situation into consideration, we change the above model with a pair of  $N$  and  $\bar{N}$  for another model with a double  $G_{st}$ -neutral field  $N_0, N_1$  and a single mirror field  $\bar{N}$ . In this case the discrete symmetry is put to  $Z_\alpha \times Z_2$  (or  $Z_{2\alpha}$ ). As suggested from superstring models, matter fields would have individual discrete charges for every generation. If  $N_0, \bar{N}, (H_u H_d)$  and  $(gg^c)$  are all even under  $Z_2$  and if  $N_1$  is odd, the superpotential  $W$  does not contain odd terms with respect to  $N_1$  due to the  $Z_2$  symmetry. Indeed, the superpotential is written as

$$\begin{aligned}
 W = & \frac{\lambda_H}{\Lambda_P^{2p}} (N_0 \bar{N})^p N_0 H_u H_d + \lambda_g N_0 g g^c + \frac{\lambda_N^{(0)}}{\Lambda_P^{2l-1}} (N_0 \bar{N})^{l+1} \\
 & + \frac{\lambda_N^{(2)}}{\Lambda_P^{2n-1}} (N_0 \bar{N})^{n-1} (N_1 \bar{N})^2 + \dots
 \end{aligned} \tag{24}$$

Since  $N_0$  has a sizable trilinear coupling to  $gg^c$  while  $N_1$  does not, it is natural that the running scalar mass squared  $m_{N_0}^2$  becomes negative but  $m_{N_1}^2$  remains positive at large energy scale. When  $m_{N_0}^2 + m_{\bar{N}}^2 < 0$ ,  $N_0$  and  $\bar{N}$  develop nonzero VEVs whereas  $\langle N_1 \rangle = 0$ . In view of the circumstances it follows that the present model exhibits hierarchical structure of the effective superpotential quite similar to that of the previous model. Therefore, under the condition  $p \geq l$  the  $\mu$ -problem and the tadpole problem are solved also in this model. It is expected that the considerations described here can be reasonably generalized to the models with more complicated

generation structure of matter fields.

In conclusion, a certain type of discrete symmetries for the underlying Planck scale theory can control the weak-scale hierarchy of the effective theory. Under the discrete symmetries the  $\mu$ -problem and the tadpole problem are closely linked together and are solved simultaneously. This is because the  $\mu$ -term and the trilinear coupling  $NH_uH_d$  in the low-energy effective theory have their origins in the common nonrenormalizable interaction. The solution is assured by the condition (15). It should be emphasized that we do not rely on some fine-tuning of parameters. Instead, it is required that doublet Higgs and colored Higgs fields reside in different irreducible representations of the gauge symmetry at the Planck scale and that they have distinct charges of the discrete symmetry. In view of the phenomenological result that in the minimal supersymmetric standard model gauge couplings are unified at the scale  $O(10^{16}\text{GeV})$  smaller than the Planck scale [11], it is tempting to find GUT-type models consistent with such particle assignments. As pointed out by the authors [12], there are such GUT-type string models. It is very interesting to construct phenomenologically viable GUT-type models which satisfies the condition (15).

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## Figure Captions

**Fig. 1** A tadpole diagram which contributes to Higgs scalar mass.  $H_u$ ,  $H_d$  and  $g$ ,  $g^c$  stand for doublet Higgs and colored Higgs fields, respectively. The  $G_{st}$ -neutral field is denoted as  $N$ .

## Table Captions

**Table I** Charges of the discrete symmetry  $Z_\alpha$  for matter superfields.  $b$  and  $c$  represent charges of the products  $(H_u H_d)$  and  $(g g^c)$ , respectively. In general, Grassmann number  $\theta$  also has a nonzero charge denoted as  $-d$ .

**Table I**

Fields	$Z_\alpha$ -charges
$N$	$a$
$\bar{N}$	$\bar{a}$
$(H_u H_d)$	$b$
$(g g^c)$	$c$
$\theta$	$-d$

This figure "fig1-1.png" is available in "png" format from:

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