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Neutron Electric Dipole Moment in Two-Higgs-Doublet Model [1]

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Abstract

The effect of the "chromo-electric" dipole moment on the electric dipole moment(EDM) of the neutron is studied in the two-Higgs-doublet model. The Weinberg's operator $O_{3g} = GG\tilde{G}$ and the operator $O_{qg} = \overline{q}\sigma\tilde{G}q$ are both investigated in the cases of $\tan\beta \gg 1$, $\tan\beta \ll 1$ and $\tan\beta \simeq 1$. The neutron EDM is considerably reduced due to the destructive contribution with two light Higgs scalars exchanges.

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1 Introduction

The electric dipole moment(EDM) of the neutron is of central importance to probe a new origin of CP violation, because it is very small in SM [1]($d_n^{SM} \simeq 10^{-30}-10^{-31} e\cdot cm$). Begining with the papers of Weinberg [2], there has been considerably renewed interest in the neutron EDM induced by CP violation of the neutral Higgs sector. Some studies [3, 4, 5] revealed the importance of the "chromo-electric" dipole moment, which arises from the three-gluon operator $GG\tilde{G}$ found by Weinberg [2] and the light quark operator $\overline{q}\sigma\tilde{G}q$ introduced by Gunion and Wyler [3], in the neutral Higgs sector. Thus, it is important to study the effect of these operators systematically in the model beyond SM. We study the contribution of above two operators to the neutron EDM in the two-Higgs-doublet model(THDM) [6]. The 3×3 mass matrix of the neutral Higgs scalars is carefully investigated in the typical three cases of $\tan\beta \gg 1$, $\tan\beta \simeq 1$ and $\tan\beta \ll 1$. In this model CP symmetry is violated through the mixing among $CP = +$ and $CP = -$ Higgs scalar states.

In order to give reliable predictions [7], one needs the improvement on the accuracy of the description of the strong-interaction hadronic effects. Chemtob [8] proposed a systematic approach which gives the hadronic matrix elements of the higher-dimension operators involving the gluon fields. We employ his model to estimate the hadronic matrix elements of the operators.

2 CP violation parameter in THDM

The simplest extension of SM is the one with the two Higgs doublets [6]. This model has the possibility of the soft CP violation in the neutral Higgs sector, which does not contribute to the flavor changing neutral current in the B, D and K meson decays. Weinberg [9] has given the unitarity bounds for the dimensionless parameters of the CP nonconservation in THDM. However, the numerically estimated values of these parameters are not always close to the Weinberg's bounds [9]. Although it is difficult to estimate the magnitudes of the CP violation parameters $\text{Im}Z_i (i = 1, 2)$ generally, we found that the neutral Higgs mass matrix is simplified in the extreme cases of $\tan\beta \ll 1$, $\tan\beta \simeq 1$ and $\tan\beta \gg 1$, in which the CP violation parameters are easily calculated. The CP violation parameters $\text{Im}Z_i^{(n)}$ are deduced to

$$\begin{aligned} \text{Im}Z_1^{(k)} &= -\frac{\tan\beta}{\cos\beta} u_1^{(k)} u_3^{(k)} , \\ \text{Im}Z_2^{(k)} &= \frac{\cot\beta}{\sin\beta} u_2^{(k)} u_3^{(k)} , \end{aligned} \tag{1}$$

where $u_i^{(k)}$ denotes the $i-$th component of the $k-$th eigenvector of the 3×3 Higgs mass matrix and $\tan\beta \equiv v_2/v_1 (v_{1(2)}$ is the vacuum expectation value of $\Phi^0_{1(2)}$ giving the masses of $d(u)$-quark sector).

In this model, Higgs potential is generally given as

$$\begin{aligned} V_H(\Phi_1, \Phi_2) &= \frac{1}{2} g_1 (\Phi_1^\dagger \Phi_1 - |v_1|^2)^2 \\ &+ \frac{1}{2} g_2 (\Phi_2^\dagger \Phi_2 - |v_2|^2)^2 \\ &+ g(\Phi_1^\dagger \Phi_1 - |v_1|^2)(\Phi_2^\dagger \Phi_2 - |v_2|^2) \\ &+ g'|\Phi_1^\dagger \Phi_2 - v_1^* v_2|^2 \\ &+ Re\{h(\Phi_1^\dagger \Phi_2 - v_1^* v_2)^2\} \\ &+ \xi[\frac{\Phi_1}{v_1} - \frac{\Phi_2}{v_2}]^\dagger [\frac{\Phi_1}{v_1} - \frac{\Phi_2}{v_2}], \end{aligned} \tag{2}$$

where the parameters satisfy the conditions [10]

$$\begin{aligned} g_1 &\geq 0, \\ g_2 &\geq 0, \\ g &> -\sqrt{g_1 g_2}, \\ g &+ g' - |h| \geq -\sqrt{g_1 g_2}, \\ \xi &\geq 0, \\ g' &-|h| + \overline{\xi} \geq 0, \end{aligned}$$

$$\bar{\xi} \quad -g \geq -\sqrt{g_1 g_2} \quad (\text{where } \bar{\xi} \equiv \frac{\xi}{|v_1 v_2|^2}). \tag{3}$$

It is noted that, in the case of MSSM, SUSY imposes the conditions on the parameters

$$\begin{aligned} g_1 = g_2 &= \tfrac{1}{4}({g_W}^2 + {g'_W}^2), \\ g &= \tfrac{1}{4}({g_W}^2 - {g'_W}^2), \\ g' &= -\tfrac{1}{2}{g_W}^2, \\ h &= 0. \end{aligned} \tag{4}$$

Here h=0 means that in MSSM CP violation is not caused throygh Higgs sector. The simplest SUSY extention from MSSM that can have CP violation in the Higgs sector is also discussed [11].

Let us estimate $u_i^{(k)}$ by studying the Higgs mass matrix $\mathbf{M^2}$ whose components are

$$\begin{aligned} M_{11}^2 &= 2g_1|v_1|^2 + g'|v_2|^2 + \frac{\xi + \mathrm{Re}(hv_1^{*2}v_2^2)}{|v_1|^2}\ , \\ M_{22}^2 &= 2g_2|v_2|^2 + g'|v_1|^2 + \frac{\xi + \mathrm{Re}(hv_1^{*2}v_2^2)}{|v_2|^2}\ , \\ M_{33}^2 &= (|v_1|^2 + |v_2|^2)\left[g' + \frac{\xi - \mathrm{Re}(hv_1^{*2}v_2^2)}{|v_1 v_2|^2}\right]\ , \\ M_{12}^2 &= |v_1 v_2|(2g + g') + \frac{\mathrm{Re}(hv_1^{*2}v_2^2) - \xi}{|v_1 v_2|}\ , \\ M_{13}^2 &= -\frac{\sqrt{|v_1|^2 + |v_2|^2}}{|v_1^2 v_2|}\mathrm{Im}(hv_1^{*2}v_2^2)\ , \\ M_{23}^2 &= -\frac{\sqrt{|v_1|^2 + |v_2|^2}}{|v_1 v_2^2|}\mathrm{Im}(hv_1^{*2}v_2^2)\ . \end{aligned} \tag{5}$$

As a phase convension, we take h to be real and

$$v_1^{*2}v_2^2 = |v_1|^2|v_2|^2\exp(2i\phi)\ . \tag{6}$$

At first, we consider the case of $\tan\beta \gg 1$ with retaining the order of $\cos\beta$ and setting $\cos^2\beta = 0$ and $\sin\beta = 1$. Then, the mass matrix becomes simple, so the eigenvectors of $\mathbf{M^2}$ in Eq.(5) are easily obtained as follows:

$$\begin{aligned} u^{(1)} &= \{\cos\beta - \epsilon\sin\beta, \quad -\sin\beta, \quad 0\}, \\ u^{(2)} &= \{\sin\beta c_\phi, \quad (\cos\beta - \epsilon\sin\beta)c_\phi, \quad -s_\phi\}, \\ u^{(3)} &= \{\sin\beta s_\phi, \quad (\cos\beta - \epsilon\sin\beta)s_\phi, \quad c_\phi\}, \end{aligned} \tag{7}$$

where $c(s)_\phi \equiv \cos(\sin)\phi$ and

$$\epsilon \simeq \frac{2(\overline{\xi} - g - g_2)}{\overline{\xi} + g' - 2g_2}\cos\beta \ . \tag{8}$$

The diagonal masses are given as

$$M_1^2 = 2g_2, \ M_2^2 = g' + \overline{\xi} + h, \ M_3^2 = g' + \overline{\xi} - h \tag{9}$$

in the $v^2 \equiv v_1^2 + v_2^2$ unit. The lightest Higgs scalar to yield CP violation is the second Higgs scalar with the mass M_2 since $\overline{\xi}$ is positive from Eq.(3) and we take h to be negative as convention. The Higgs scalar with M_1 does not contribute to CP violation because of $u_3^{(1)} = 0$. The absolute values of g' is expected to be $O(1)$, but h seems to be small as estimated in some works [12, 13]. For example Froggatt et al. give the numerical values for the parameters in the case of $\tan\beta \gg 1$ by using infrared fixed point analysis through the renormalization group equations as

$$\begin{aligned} &g_1 \simeq 0.96, \ g_2 \simeq 0.88, \ g \simeq 0.82 \\ &\qquad g' \simeq -1.20, \ h \simeq -0.09. \end{aligned} \tag{10}$$

Therefore, the masses M_2 and M_3 may be almost degenerated. Then, CP violation is reduced by the cancellation between the two different Higgs exchange contributions $\mathrm{Im}Z_i^{(2)}$ and $\mathrm{Im}Z_i^{(3)}$ since $u_i^{(2)}u_3^{(2)}$ and $u_i^{(3)}u_3^{(3)}$(i=1,2) have same magnitudes with opposite signs. Thus, it is noted that the lightest single Higgs exchange approximation gives miss-leading of CP violation in the case of $\tan\beta \gg 1$.

For $\mathrm{Im}Z_1$, our result reaches the Weinberg bound, but for $\mathrm{Im}Z_2$ the our calculated value is suppressed compared with the Weinberg bound in the order of $1/\tan\beta$.

CP violation in the case of $\tan\beta \ll 1$ is similar to the one of $\tan\beta \gg 1$. For $\mathrm{Im}Z_2$, our numerical result reaches the Weinberg bound, while for $\mathrm{Im}Z_1$ the calculated value is suppressed from the Weinberg bound in the order of $\tan\beta$. The relative sign between $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$ is just the same as in the case of $\tan\beta \gg 1$.

The last case to be considered is of $\tan\beta \simeq 1$. In this mass matrix, the off diagonal components are very small compared to the diagonal ones because $g_1 \simeq g_2$ is suggested by some analyses [12, 13] and h is also small as in the case of $\tan\beta \gg 1$. We can calculate $\mathrm{Im}Z_i$ by fixing both values of h and M_2/M_3. For both $\mathrm{Im}Z_2$ and $\mathrm{Im}Z_1$, the calculating values are roughly 1/3 of the Weinberg bounds. The relative sign between $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$ is opposite.

3 Formulation of the neutron EDM

The low energy CP-violating interaction is described by an effective Lagrangian,

$$L_{CP} = \sum_i C_i(M,\mu) O_i(\mu) \ , \tag{11}$$

where O_i are the three gluon operator with the dimension six and the quark-gluon operator with the dimension five as follows:

$$\begin{aligned} O_{qg}(x) &= -\frac{g_s^3}{2}\bar{q}\sigma_{\mu\nu}\tilde{G}^{\mu\nu}q \ , \\ O_{3g}(x) &= -\frac{g_s^3}{3} f^{abc}\tilde{G}^a_{\mu\nu}G^b_{\mu\alpha}G^c_{\nu\alpha} \ , \end{aligned} \tag{12}$$

where q denotes u, d or s quark. The QCD corrected coefficients C_i are given by the two-loop calculations in Refs. [2, 3]. The coefficients C_i are given as

$$\begin{aligned} C_{ug} &= \frac{\sqrt{2}G_F m_u}{64\pi^4}\{f(\frac{m_t^2}{m_H^2}) + g(\frac{m_t^2}{m_H^2})\}\mathrm{Im}Z_2(\frac{g_s(\mu)}{g_s(M)})^{-\frac{74}{23}}, \\ C_{dg} &= \frac{\sqrt{2}G_F m_d}{64\pi^4}\{f(\frac{m_t^2}{m_H^2})\tan^2\beta\mathrm{Im}Z_2 \\ &- g(\frac{m_t^2}{m_H^2})\cot^2\beta\mathrm{Im}Z_1\}(\frac{g_s(\mu)}{g_s(M)})^{-\frac{74}{23}}, \end{aligned}$$

$$C_{3g} = \frac{\sqrt{2}G_F}{(4\pi)^4}\mathrm{Im}Z_2 h(\frac{m_t^2}{m_H^2})(\frac{g_s(\mu)}{g_s(M)})^{-\frac{108}{23}}, \tag{13}$$

where the functions $f(x), g(x), h(x)$ are deduced from loop integral as given in Refs. [2, 3].

For the strong interaction hadronic effect, the systematic technique has been developed by Chemtob [8] in the operator with the higher-dimension involving the gluon fields. The hadronic matrix elements of the two operators are approximated by the intermediate states with the single nucleon pole and the nucleon plus one pion. Then, the nucleon matrix elements are defined as

$$\begin{aligned} \langle N(P)|O_i(0)|N(P)\rangle &= A_i\overline{U}(P)i\gamma_5 U(P), \\ \langle N(P')|O_i|N(P)\pi(k)\rangle &= B_i\overline{U}(P')\tau^a U(P)\ , \end{aligned} \tag{14}$$

where $U(P)$ is the normalized nucleon Dirac spinors with the four momuntum P. Using A_i and $B_i(i = ug, dg, sg, 3g)$, the neutron EDM, d_n^γ, are written as

$$d_n^\gamma = \frac{e\mu_n}{2m_n^2}\sum_i C_i A_i + F(g_{\pi NN})\sum_i C_i B_i\ , \tag{15}$$

where μ_n is the neutron anomalous magnetic moment. The $F(g_{\pi NN})$ was given by calculating the pion and nucleon loop corrections using the chiral Lagrangian for $N\pi\gamma$ [8]. The coefficients A_i and B_i were given by the large N_c current algebra and the η_0 meson dominance [8].

4 Numerical results of the neutron EDM

Let us begin with discussing the numerical results in the case of $\tan\beta \gg 1$. The contributions of O_{ug} and O_{3g} are are negligibly small because the CP violation parameters are roughly estimated as

$$\begin{aligned} \mathrm{Im}Z_2^{(2)} \simeq -\mathrm{Im}Z_2^{(3)} \simeq \frac{1}{\tan^2\beta} \ll \mathrm{Im}Z_1^{(2,3)}, \\ \mathrm{Im}Z_1^{(3)} \simeq -\mathrm{Im}Z_1^{(3)} \simeq \frac{1}{2}\tan^2\beta. \end{aligned} \tag{16}$$

The main contribution follows from the one of $O_{dg}+O_{sg}$, in which the operator O_{sg} is dominant due to the s-quark mass. The coefficient C_{sg} is

$$\begin{aligned} C_{sg} &= (const.) \times m_s \{ f(\tfrac{m_t^2}{m_{H_2}^2}) - f(\tfrac{m_t^2}{m_{H_3}^2}) \\ &\quad - \tfrac{1}{2} g(\tfrac{m_t^2}{m_{H_2}^2}) + \tfrac{1}{2} g(\tfrac{m_t^2}{m_{H_3}^2}) \}. \end{aligned} \tag{17}$$

As the mass difference of these two Higgs scalar masses becomes smaller, the neutron EDM is considerably reduced since the second Higgs scalar exchange contributes in the opposite sign to the lightest Higgs scalar one as shown in the above equation. Thus, it is found that the second lightest Higgs scalar also significantly contributes to CP violation.

In the case of $\tan\beta \ll 1$, the contributions of O_{ug} and O_{3g} become very large due to the large ImZ_2. However, these contribute to the neutron EDM in opposite signs, so they almost cancel each other. The remaining contribution is the one of $O_{dg}+O_{sg}$.

In the case of $\tan\beta \simeq 1$, the dominant contribution is the one of $O_{dg}+O_{sg}$. In both regions of the large and small m_{H2}/m_{H3}, the predicted neutron EDM is reduced. At $m_{H2}/m_{H3} \simeq 1$, the cancellation mechanism by the second lightest Higgs scalar operates well, while around $m_{H2}/m_{H3} \simeq 0$, the large mass difference of the two Higgs scalars leads to the small mixing between the scalar and pseudscalar Higgs bosons.

5 Summary

We have studied the effects of the four operators O_{ug}, $O_{dg}+O_{sg}$ and O_{3g} on the neutron EDM. The contribution of O_{sg} dominates over that of other operators. Moreover, the contributions of O_{ug} and O_{3g} cancel out each other due to their opposite signs. This qualitative situation does not depend on the detail of the strong interaction hadronic model. Thus, the Weinberg's three gluon operator is not a main source of the neutron EDM in THDM. The CP violation mainly follows from the two light neutral Higgs scalar exchanges. Since these two exchange contributions are of opposite signs, the

CP violation is considerably reduced if the mass difference of the two Higgs scalars is small. Since our predicted neutron EDM lies around the present experimental bound, its experimental improvement reveal the new physics beyond SM. The present upper limit for d_n^γ is $8\times10^{-26} e\cdot cm$ which was given at the 26th ICHEP. Historically to reduce one order of magnitude for upper limit experimentally, it has taken almost 10 years. We hope that the rapid experimental reduction of upper limit will be performed and that the finite value will be reported at the close ICHEP.

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