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Neutron Electric Dipole Moment in Two Higgs Doublet Model

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ABSTRACT

We study the effect of the "chromo-electric" dipole moment on the electric dipole moment(EDM) of the neutron in the two Higgs doublet model. We systematically investigate the Weinberg's operator $O_{3g} = GG\tilde{G}$ and the operator $O_{qg} = \overline{q}\sigma\tilde{G}q$, in the cases of $\tan\beta \gg 1$, $\tan\beta \ll 1$ and $\tan\beta \simeq 1$. It is shown that O_{sg} gives the main contribution to the neutron EDM compared to the other operators, and also that the contributions of O_{ug} and O_{3g} cancel out each other. It is pointed out that the inclusion of second lightest neutral Higgs scalar adding to the lightest one is of essential importance to estimate the neutron EDM. The neutron EDM is considerably reduced due to the destructive contribution with each other if the mass difference of the two Higgs scalars is of the order $O(50\text{GeV})$.

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1. Introduction

The physics of CP violation has attracted one's attention in the circumstance that the B-factory will go on in the near future. In the experiments of the B decay asymmetry, the central subject is the test of the standard Kobayashi-Maskawa model(SM)[1] as an origin of CP violation. On the other hand, the electric dipole moment(EDM) of the neutron is of central importance to probe a new origin of CP violation, because it is very small in SM. Begining with the papers of Weinberg[2], there has been considerable renewed interest in the neutron EDM induced by CP nonconservation of the neutral Higgs sector. Some studies[3,4,5] revealed numerically the importance of the "chromo-electric" dipole moment, which arises from the three-gluon operator $GG\tilde{G}$ by found Weinberg[2] and the light quark operator $\overline{q}\sigma\tilde{G}q$ introduced by Gunion and Wyler[3], in the neutral Higgs sector. Thus, it is important to study the effect of these operators systematically in the model beyond SM. In this paper, we study the contribution of above two operators to the neutron EDM in the two Higgs doublets model(THDM)[6]. The 3×3 mass matrix of the neutral Higgs scalars is carefully investigated in the typical three cases of $\tan\beta \gg 1$, $\tan\beta \simeq 1$ and $\tan\beta \ll 1$, where $\tan\beta \equiv |v_2/v_1|$ with $v_i \equiv \langle\phi_i^0\rangle_{vac}$ and ϕ_1 and ϕ_2 couple with down- and up-quark sectors, respectively. In these restricted regions of $\tan\beta$, the Higgs mass matrix becomes very simple, and then we can easily estimate the CP violation parameters of the neutral Higgs sector, which lead to the neutron EDM in THDM. We found that the neutron EDM follows mainly from the two light neutral Higgs scalar exchanges. Due to the opposite signs of the two contributions, the neutron EDM is considerably reduced if the mass difference of the two Higgs scalars is in the order of $O(50\mathrm{GeV})$.

In order to give reliable predictions, one needs the improvement on the accuracy of the description of the strong-interaction hadronic effects. Weinberg employed the "naive dimensional analyse"(NDA) as developed by Georgi and Manohar[7] in computing the effect of the $GG\tilde{G}$ operator on the neutron. However, this method admittedly provides at best the order-of-magnitude estimation. Moreover, when gluon fields are present, there occurs an indeterminable factor of 4π, which depends on whether one associates a factor g_s or $4\pi g_s$ with each gluon field factor in the interaction Lagrangian[3]. Recently, Chemtob[8] proposed a systematic approach which gives the hadronic matrix elements of the higher-dimension operators involving the gluon fields by using the large N_c current-algebra. In his model, the hadronic matrix elements of the operators are approximated by the intermediate states with the single nucleon pole and the nucleon plus one pion. So, this approach may be a realistic one. We employ his model to get the hadronic matrix elements of the relevant operators in this work. The comparision between results by this approach and NDA will be discussed briefly in the last section.

In section 2, the neutral Higgs mass matrix is analyzed and then the magnitudes of the CP violation factors $\mathrm{Im}Z_i$ are estimated. In section 3, the formulation of the neutron EDM with the hadronic matrix elements of the CP violating operators are discussed. Section 4 is devoted to the numerical results of the neutron EDM and some remarks and conclusion are given in section 5.

2. CP violation parameter in THDM

The simple extension of SM is the one with the two Higgs doublets[6]. This model has the possibility of the soft CP violation in the neutral Higgs sector, which does not contribute to the flavor changing neutral current in the B, D and K meson decays. Weinberg[9] has given the unitarity bounds for the dimensionless parameters of the CP

nonconservation in THDM. However, values of these parameters are not always close to the Weinberg's bounds[9]. Actually, the CP violation parameter $\mathrm{Im}Z_1$(this definition is given later) is suppressed by $1/\tan\beta$ compared with the Weinberg's bound at the large $\tan\beta$ as pointed out by Barr[5]. Chemtob[10] has predicted CP violation parameters by the use of the renormalization approach under the assumption that the coupling constants of the Yukawa couplings and self-coupling scalar mesons interactions reach infrared fixed points at the electroweak scale. This infrared fixed points approach either leads to the large top quark mass $m_t \sim 230$GeV, which is unfavourable to the recent electroweak precision test, or leads to the existence of the unobserved fourth generation quarks. Thus, it is difficult to estimate the reliable magnitudes of the CP violation parameters $\mathrm{Im}Z_i (i=1,2)$. However, we found that the Higgs mass matrix is simplified in the extreme cases of $\tan\beta \ll 1$, $\tan\beta \simeq 1$ and $\tan\beta \gg 1$, in which the CP violation parameters are easily calculated.

The CP violation will occur via the scalar-pseudoscalar interference terms involving the imaginary parts of the scalar meson fields normalization constants, Z_i, which are column vectors in the neutral Higgs scalar vector space, defined in terms of the tree level approximation to the two-point function as follows:

$$\begin{aligned}&\left[\ \tfrac{1}{v_1^2}\langle\phi_1^0\phi_1^0\rangle_q, \quad \tfrac{1}{v_2^2}\langle\phi_2^0\phi_2^0\rangle_q, \quad \tfrac{1}{v_1 v_2}\langle\phi_2^0\phi_1^0\rangle_q, \quad \tfrac{1}{v_1^* v_2}\langle\phi_2^0\phi_1^{*0}\rangle_q\ \right] \\ &= \sum_{n=1}^{3}\frac{\sqrt{2}G_F}{q^2-m_{Hn}^2}[\ Z_1^{(n)}, \quad Z_2^{(n)}, \quad \tilde{Z}_0^{(n)}, \quad Z_0^{(n)}\ ]\ ,\end{aligned} \tag{1}$$

where $v_i \equiv \langle\phi_i^0\rangle_{vac}$. The CP violation factors $\mathrm{Im}Z_i^{(n)}$ are deduced to

$$\begin{aligned}\mathrm{Im}Z_2^{(k)} &= \frac{1}{\tan\beta\sin\beta}u_2^{(k)}u_3^{(k)}\ , \quad \mathrm{Im}Z_1^{(k)} = -\frac{\tan\beta}{\cos\beta}u_1^{(k)}u_3^{(k)}\ , \\ \mathrm{Im}\tilde{Z}_0^{(k)} &= \frac{1}{2}\left(\frac{1}{\sin\beta}u_1^{(k)} - \frac{1}{\cos\beta}u_2^{(k)}\right)u_3^{(k)}\ , \quad \mathrm{Im}Z_0^{(k)} = \frac{1}{2}\left(\frac{1}{\sin\beta}u_1^{(k)} + \frac{1}{\cos\beta}u_2^{(k)}\right)u_3^{(k)}\ ,\end{aligned} \tag{2}$$

where $u_i^{(k)}$ denotes the $i-$th component of the $k-$th normalized eigenvector of the

Higgs mass matrix. Let us estimate $u_i^{(k)}$ by studying the symmetric Higgs mass matrix $\mathbf{M^2}$ whose components are

$$\begin{aligned}
M_{11}^2 &= 2g_1|v_1|^2 + g'|v_2|^2 + \frac{\xi + Re(hv_1^{*2}v_2^2)}{|v_1|^2} \ , \\
M_{22}^2 &= 2g_2|v_2|^2 + g'|v_1|^2 + \frac{\xi + Re(hv_1^{*2}v_2^2)}{|v_2|^2} \ , \\
M_{33}^2 &= (|v_1|^2 + |v_2|^2)\left[g' + \frac{\xi - Re(hv_1^{*2}v_2^2)}{|v_1v_2|^2}\right] \ , \\
M_{12}^2 &= |v_1v_2|(2g+g') + \frac{Re(hv_1^{*2}v_2^2) - \xi}{|v_1v_2|} \ , \\
M_{13}^2 &= -\frac{\sqrt{|v_1|^2+|v_2|^2}}{|v_1^2v_2|}\mathrm{Im(hv_1^{*2}v_2^2)} \ , \\
M_{23}^2 &= -\frac{\sqrt{|v_1|^2+|v_2|^2}}{|v_1v_2^2|}\mathrm{Im(hv_1^{*2}v_2^2)} \ ,
\end{aligned} \tag{3}$$

which are derived from the Higgs potential

$$\begin{aligned}
V &= \frac{1}{2}g_1(\phi_1^\dagger\phi_1 - |v_1|^2)^2 + \frac{1}{2}g_2(\phi_2^\dagger\phi_2 - |v_2|^2)^2 \\
&+ g(\phi_1^\dagger\phi_1 - |v_1|^2)(\phi_2^\dagger\phi_2 - |v_2|^2) \\
&+ g'|\phi_1^\dagger\phi_2 - v_1^*v_2|^2 + Re[h(\phi_1^\dagger\phi_2 - v_1^*v_2)^2] \\
&+ \xi\left[\frac{\phi_1}{v_1} - \frac{\phi_2}{v_2}\right]^\dagger\left[\frac{\phi_1}{v_1} - \frac{\phi_2}{v_2}\right] \ .
\end{aligned} \tag{4}$$

As a phase convension, we take h to be real and

$$v_1^{*2}v_2^2 = |v_1|^2|v_2|^2\exp(2i\phi) \ . \tag{5}$$

Now, the Higgs mass matrix $\mathbf{M^2}$ is rotated so as to make the (1,3)(and then (3,1)) component zero by the orthogonal matrix $\mathbf{U_0}$ as

$$\mathbf{U_0} = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \ . \tag{6}$$

Then, the transformed matrix $\mathbf{M'^2} = \mathbf{U_0^t M^2 U_0}$ is given as

$$\begin{aligned}
M'^2_{11} &= 2g_1\cos^4\beta + 2g_2\sin^4\beta + 4(\overline{\xi} - g)\sin^2\beta\cos^2\beta \ , \\
M'^2_{22} &= 2(g_1 + g_2 + 2g - 2\overline{\xi})\sin^2\beta\cos^2\beta \ + g' + \overline{\xi} + h\cos 2\phi \ , \\
M'^2_{33} &= g' + \overline{\xi} - h\cos 2\phi \ , \\
M'^2_{12} &= \sin\beta\cos\beta\left[\cos 2\beta(2g - 2\overline{\xi} + g_1 + g_2) + g_1 - g_2\right] \ , \\
M'^2_{13} &= 0 \ , \\
M'^2_{23} &= -h\sin 2\phi \ ,
\end{aligned} \tag{7}$$

in the $v^2 \equiv |v_1|^2 + |v_2|^2$ unit and the parameter $\overline{\xi}$ is defined as $\overline{\xi} = \xi/|v_1 v_2|^2$. This matrix cannot be diagonalized in the analytic form generally, unless special relations among the parameters of the mass matrix are satisfied. The parameters are only constrained by the positivity condition as follows[10,11]:

$$g_1 > 0 \ , \quad g_2 > 0 \ , \quad h < 0 \ , \quad h + g' < 0 \ , \quad g + g' + h > -\sqrt{g_1 g_2} \ . \tag{8}$$

However, we can simply diagonalize the Higgs mass matrix in the extreme cases of $\tan\beta \gg 1$, $\tan\beta \simeq 1$ and $\tan\beta \ll 1$.

At first, we consider the case of $\tan\beta \gg 1$. By retaining the order of $\cos\beta$ and by setting $\cos^2\beta = 0$, $\sin\beta = 1$, the mass matrix becomes

$$\begin{pmatrix} 2g_2 & 2\cos\beta(\overline{\xi} - g - g_2) & 0 \\ 2\cos\beta(\overline{\xi} - g - g_2) & g' + \overline{\xi} + h\cos 2\phi & -h\sin 2\phi \\ 0 & -h\sin 2\phi & g' + \overline{\xi} - h\cos 2\phi \end{pmatrix} . \tag{9}$$

In the limit of $\cos\beta = 0$, this mass matrix is diagonalized by only rotating ϕ on the (2-3) plane. However, due to the non-vanishing tiny M'^2_{12} component, this rotation is slightly deviated from the (2-3) plane. The orthogonal matrix $\mathbf{U_1}$ to diagonalize the

Higgs mass matrix of Eq.(9) is approximately obtained as:

$$\mathbf{U_1} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & \epsilon\cos\phi & 0 \\ -\epsilon\cos\phi & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \epsilon\sin\phi \\ 0 & 1 & 0 \\ -\epsilon\sin\phi & 0 & 1 \end{pmatrix} , \quad (10)$$

where, neglecting $h\cos 2\phi$,

$$\epsilon \simeq \frac{2(\overline{\xi} - g - g_2)}{\overline{\xi} + g' - 2g_2} \cos\beta \ . \quad (11)$$

Then, the eigenvectors of $\mathbf{M^2}$ in Eq.(1) are

$$\begin{aligned} u^{(1)} &= \{\cos\beta - \epsilon\sin\beta, \quad -\sin\beta, \quad 0\} , \\ u^{(2)} &= \{\sin\beta\cos\phi, \quad (\cos\beta - \epsilon\sin\beta)\cos\phi, \quad -\sin\phi\} , \\ u^{(3)} &= \{\sin\beta\sin\phi, \quad (\cos\beta - \epsilon\sin\beta)\sin\phi, \quad \cos\phi\} , \end{aligned} \quad (12)$$

with the order of $O(\cos^2\beta)$ being neglected. The diagonal masses are given as

$$M_1^2 = 2g_2 + O(\cos^2\beta), \quad M_2^2 = g' + \overline{\xi} + h + O(\cos^2\beta), \quad M_3^2 = g' + \overline{\xi} - h + O(\cos^2\beta) , \quad (13)$$

in the v^2 unit. The lightest Higgs scalar to yield CP violation is the second Higgs scalar with the mass M_2 since h is negative and $\overline{\xi}$ is positive. The Higgs scalar with M_1 does not contribute to CP violation because of $u_3^{(1)} = 0$. The absolute values of g' is expected to be $O(1)$, but h seems to be small as estimated in some works[11]. Therefore, the masses M_2 and M_3 may be almost degenerated. Then, CP violation is reduced by the cancellation between the two different Higgs exchange contributions $\mathrm{ImZ_i^{(2)}}$ and $\mathrm{ImZ_i^{(3)}}$ since $u_i^{(2)} u_3^{(2)}$ and $u_i^{(3)} u_3^{(3)}$(i=1,2) have same magnitudes with opposite signs. Thus, it is noted that the lightest single Higgs exchange approximation gives miss-leading of CP violation in the case of $\tan\beta \gg 1$.

In order to get the magnitudes of $u_2^{(2,3)}$, we estimate ϵ, which depends on the value of $\overline{\xi}$. The parameter $\overline{\xi}$ is determined by the charged Higgs mass as follows:

$$M^{\pm 2} = \overline{\xi} v^2 \ . \quad (14)$$

We have already studied the charged Higgs scalar effect in THDM through the inclusive decay $B \to X_s \gamma$[12], as to which the upper bound of the branching ratio was recently given by the CLEO collaboration[13]. We obtained 300GeV for the lower bound of the charged Higgs scalar mass in the case of $m_t = 150$GeV and $m_b = 5$GeV. This lower bound means $\overline{\xi} > 3$. In the limit of the large $\overline{\xi}$ with retaining other parameters to be $O(1)$, $\epsilon/\cos\beta$ reachs 2 as seen in Eq.(11). Actually, g_1, g_2, g and $|g'|$ are around 1 in some numerical studies[11]. Then, if we take $M^{\pm} = 400(350)$GeV, which corresponds to $\overline{\xi} = 5(4)$, the value of $\epsilon/\cos\beta$ becomes 3(4). In the following calculations, we fix to be $\epsilon = 3\cos\beta$. By use of these resulting $u_i^{(2)}$ values, we can calculate the CP violation factors $\mathrm{Im}Z_i = \mathrm{Im}Z_i^{(2)}$. We show the numerical results together with the Weinberg bounds for $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$ in Figs.1(a) and 1(b), where $\phi = \pi/4$ is taken. Although the Weinberg bounds give nothing for these signs, our estimates determine the relative sign between $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$. For $\mathrm{Im}Z_1$, our result reachs the Weinberg bound, but for $\mathrm{Im}Z_2$ the our calculated value is suppressed compared with the Weinberg bound in the order of $1/\tan\beta$.

$\boxed{\textbf{Figs.1(a)} \sim \textbf{1(f)}}$

CP violation in the case of $\tan\beta \ll 1$ is similar to the one of $\tan\beta \gg 1$. By retaining the order of $\sin\beta$ and by setting $\sin^2\beta = 0$, $\cos\beta = 1$, the mass matrix becomes

$$\begin{pmatrix} 2g_1 & -2\sin\beta(\overline{\xi} - g - g_1) & 0 \\ -2\sin\beta(\overline{\xi} - g - g_1) & g' + \overline{\xi} + h\cos 2\phi & -h\sin 2\phi \\ 0 & -h\sin 2\phi & g' + \overline{\xi} - h\cos 2\phi \end{pmatrix} . \tag{15}$$

Except for replacing g_2 with g_1 and $\cos\beta$ with $-\sin\beta$, the mass matrix is the same one as of Eq.(9) in the case of $\tan\beta \ll 1$. The eigenvectors are easily obtained as follows:

$$
\begin{aligned}
u^{(1)} &= \{\cos\beta, \quad -(\sin\beta + \epsilon'\cos\beta), \quad 0\} , \\
u^{(2)} &= \{(\epsilon' + \sin\beta)\cos\phi, \quad \cos\beta\cos\phi, \quad -\sin\phi\} , \\
u^{(3)} &= \{(\epsilon' + \sin\beta)\cos\phi, \quad \cos\beta\sin\phi, \quad \cos\phi\} ,
\end{aligned}
\tag{16}
$$

with the order of $O(\sin^2\beta)$ being neglected. The ϵ' parameter is defines as

$$
\epsilon' = -\frac{2(\overline{\xi} - g - g_1)}{\overline{\xi} + g' - 2g_1 + h\cos 2\phi}\sin\beta , \tag{17}
$$

which is taken to be $\epsilon' = -3\sin\beta$ as discussed in Eq.(11). Taking $u_i^{(2)}$ as the eigenvector of the lightest Higgs scalar, we show $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$ in Figs.1(c) and 1(d). For $\mathrm{Im}Z_2$, our result reachs the Weinberg bound, while for $\mathrm{Im}Z_1$ the calculated value is suppressed from the Weinberg bound in the order of $\tan\beta$. The relative sign between $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$ is just same as in the case of $\tan\beta \gg 1$.

The last case to consider is of $\tan\beta \simeq 1$. Setting $\cos 2\beta = 0$, we get the Higgs mass matrix as

$$
\begin{pmatrix}
\frac{1}{2}g_1 + \frac{1}{2}g_2 + \overline{\xi} - g & \frac{1}{2}(g_1 - g_2) & 0 \\
\frac{1}{2}(g_1 - g_2) & \frac{1}{2}g_1 + \frac{1}{2}g_2 + g + g' + h\cos 2\phi & -h\sin 2\phi \\
0 & -h\sin 2\phi & g' + \overline{\xi} - h\cos 2\phi
\end{pmatrix} . \tag{18}
$$

The off diagonal components are very small compared to the diagonal ones because $g_1 \simeq g_2$ is suggested by some analyses[11] and h is small. Then, we get the approximate eigenvectors as follows:

$$
\begin{aligned}
u^{(1)} &= \{\cos\beta - \sin\beta\sin\theta_{12}\cos\theta_{23}, \quad -\sin\beta - \cos\beta\sin\theta_{12}\cos\theta_{23}, \quad \sin\theta_{12}\sin\theta_{23}\}, \\
u^{(2)} &= \{\sin\beta\cos\theta_{23} + \cos\beta\sin\theta_{12}, \quad \cos\beta\cos\theta_{23} - \sin\beta\sin\theta_{12}, \quad -\sin\theta_{23}\} , \\
u^{(3)} &= \{(\sin\beta\sin\theta_{23}, \quad \cos\beta\sin\theta_{23}, \quad \cos\theta_{23}\} ,
\end{aligned}
\tag{19}
$$

where

$$\tan 2\theta_{12} = \frac{g_2 - g_1}{\overline{\xi} - 2g - g' - h\cos 2\phi} \ ,$$
$$\tan 2\theta_{23} = \frac{4h\sin 2\phi}{g_1 + g_2 + 2g - 2\overline{\xi} + 4h\cos 2\phi} \simeq \frac{2h\sin 2\phi}{M_2^2 - M_3^2}v^2 \ . \qquad (20)$$

The Higgs scalar mass M_1 is expected to be the heaviest one and the M_2 to be the lightest one because of $\overline{\xi} > 3.0$ and $g_1 \sim g_2 \sim g \sim |g'| \simeq O(1)$. We estimate the effect of CP violation by considering the Higgs scalar with M_2 being lightest one and then add the effect of the one with M_3. Since θ_{12} is expected to be of $O(10^{-2})$[11], we neglect terms with $\sin\theta_{12}$ in Eq.(19). We can calculate the CP violation parameters $\mathrm{Im}Z_i$ by fixing both values of h and M_2/M_3. We show $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$ in Figs.1(e) and 1(f) taking $M_2 = 200$GeV, $M_3 = 250$GeV and $h = -0.1$. For both $\mathrm{Im}Z_2$ and $\mathrm{Im}Z_1$, the calculating values are roughly 1/3 of the Weinberg bounds. The relative sign between $\mathrm{Im}Z_1$ and $\mathrm{Im}Z_2$ is opposite to the one in the cases of $\tan\beta \gg 1$ and $\tan\beta \ll 1$.

3. Formulation of the neutron EDM

The low energy CP-violating interaction is described by an effective Lagrangian L_{CP}, which is generally decomposed into the local composite operators O_i of the quarks and gluons fields,

$$L_{CP} = \sum_i C_i(M,\mu) O_i(\mu) \ . \qquad (21)$$

Some authors pointed out[3,8] that the three gluon operator with the dimension six and the quark-gluon operator with the dimension five dominate EDM of the neutron in THDM. So, we study the effect of these two operators on the neutron EDM. Various techniques have been developed to estimate the strong-interaction hadronic effects[7,8,14]. The simplest one is the NDA approach[7], but it provides at best the order-of-magnitude estimates. The systematic technique has been given by Chemtob[8] for the case of the operator with the higher-dimension involving the gluon fields. We employ his technique to get the hadronic matrix elements of the operators.

Let us define the following operators:

$$O_{qg}(x) = -\frac{g_s}{2}\overline{q}\sigma_{\mu\nu}\tilde{G}^{\mu\nu}q \ , \qquad O_{3g}(x) = -\frac{g_s^3}{3}f^{abc}\tilde{G}^a_{\mu\nu}G^b_{\mu\alpha}G^c_{\nu\alpha} \ , \tag{22}$$

where q denotes u, d or s quark. The QCD corrected coefficients are given by the two-loop calculations[2,3] as follows:

$$\begin{aligned}
C_{ug} &= -\frac{\sqrt{2}G_F m_u(\mu)}{128\pi^4}g_s^2(\mu)[f(z_t)+g(z_t)]\mathrm{ImZ_2}\left(\frac{\mathrm{g_s}(\mu)}{\mathrm{g_s(M)}}\right)^{-\frac{74}{23}} \ , \\
C_{dg} &= -\frac{\sqrt{2}G_F m_d(\mu)}{128\pi^4}g_s^2(\mu)[f(z_t)\tan^2\beta\mathrm{ImZ_2} - \mathrm{g(z_t)}\cot^2\beta\mathrm{ImZ_1}]\left(\frac{\mathrm{g_s}(\mu)}{\mathrm{g_s(M)}}\right)^{-\frac{74}{23}} \ , \\
C_{3g} &= \frac{\sqrt{2}G_F}{256\pi^4}h(z_t)\mathrm{ImZ_2}\left(\frac{\mathrm{g_s}(\mu)}{\mathrm{g_s(M)}}\right)^{-\frac{108}{23}} \ ,
\end{aligned} \tag{23}$$

where $z_t = (m_t/m_H)^2$ and we omitt the upper-indices (k) defined in Eq.(2). The function $f(z_t)$, $g(z_t)$ and $h(z_t)$ are the two-loop integral function, which are defined in Refs.[4,5,15]. The C_{sg} coefficient is same as C_{dg} except for the quark mass. In our practical calculation, the modification to account for the passage through the b and c quarks thresholds involves the replacement

$$\left(\frac{g_s(\mu)}{g_s(M)}\right)^{\frac{n}{23}} \longrightarrow \left(\frac{g_s(m_b)}{g_s(M)}\right)^{\frac{n}{23}}\left(\frac{g_s(m_c)}{g_s(m_b)}\right)^{\frac{n}{25}}\left(\frac{g_s(\mu)}{g_s(m_c)}\right)^{\frac{n}{27}} \ . \tag{24}$$

The hadronic matrix elements of the two operators are approximated by the intermediate states with the single nucleon pole and the nucleon plus one pion. Then, the nucleon matrix elements are defined as

$$\begin{aligned}
\langle N(P)|O_i(0)|N(P)\rangle &= A_i\overline{U}(P)i\gamma_5 U(P) \ , \\
\langle N(P')|O_i|N(P)\pi(k)\rangle &= B_i\overline{U}(P')\tau^a U(P) \ ,
\end{aligned} \tag{25}$$

where $U(P)$ is the normalized nucleon Dirac spinors with the four momuntum P. Using A_i and $B_i (i = ug, dg, sg, 3g)$, the neutron EDM, d_n^γ, are written as

$$d_n^\gamma = \frac{e\mu_n}{2m_n^2} \sum_i C_i A_i + F(g_{\pi NN}, m_n, m_\pi) \sum_i C_i B_i \ , \tag{26}$$

where μ_n is the neutron anomalous magnetic moment. The function $F(g_{\pi NN}, m_n, m_\pi)$ was derived by calculating the pion and nucleon loop corrections using the chiral Lagrangian for the coupled $N\pi\gamma$ and in given in Appendix A of Ref.[8]. Here, the dimensional regularization with the standard $\overline{MS}$ scheme is used for defining the finite parts of the divergent integrals. The coefficients A_i and B_i were given by the use of the large N_c current algebra and the η_0 meson dominance[8]. Then, we have

$$A_i = f_i g_{\eta_0 NN} \ , \qquad\qquad B_i = -\frac{4(m_u + m_d) a_1 f_i}{F_\pi F_0} \ , \tag{27}$$

with $a_1 = -(m_{\Sigma^0} - m_\Sigma)/(2m_s - m_u - m_d) \simeq -0.28$ and $F_\pi = \sqrt{2/3} F_0 = 0.186\text{GeV}$, where f_i is defined as

$$\langle \eta_0(q) | O_i(0) | 0 \rangle \equiv f_i q^2 \ . \tag{28}$$

The values of f_i were derived by using QCD sum rules as follows[8]:

$$f_{qg} = -0.346\text{GeV}^2 \ , \qquad\qquad f_{3g} = -0.842\text{GeV}^3 \ , \tag{29}$$

where f_{qg} denotes the flavor singlet coupling.

Now, we can calculate the neutron EDM. Our inputs parameters are

$$\begin{aligned} &\Lambda_{QCD} = 0.26\text{GeV} \ , \qquad (m_u, m_d, m_s) = (5.6, 9.9, 200)\text{MeV} \ , \qquad \mu = m_n \ , \\ &M = m_t = 150\text{GeV} \ , \qquad g_{\pi NN} = 13.5 \ , \qquad g_{\eta_0 NN} = 0.892 \ . \end{aligned} \tag{30}$$

Here, it is useful to comment on the value of μ. As the smaller μ is taken, the QCD suppression factor increases, and then, the predicted neutron EDM decreases. Although we do not have the reliable principle to fix μ in the leading-log approximation of QCD, we tentatively take $\mu = m_n$, which leads $\alpha_s(\mu) = 0.54$. If we take $\mu = 0.6$GeV, which gives rather large $\alpha_s(\mu) = 0.83$, as used by Chemtob[8], our predicted neutron EDM will be reduced by a factor $2 \sim 4$.

4. Numerical analyses of the neutron EDM

We show the numerical results in this section. Since the CP violation parameters $\mathrm{ImZ_i}$ have been estimated in the three cases of $\tan\beta$, the neutron EDM is predicted for each case of $\tan\beta$. Since our results are proportional to $\sin 2\phi$, we take the maximal case $\phi = \pi/4$ in showing the numerical results. We show the contribution of the four operators O_{ug}, $O_{dg} + O_{sg}$ and O_{3g} on the neutron EDM, respectively. At first, we show the predicted neutron EDM in the region of $5 \leq \tan\beta \leq 10$, which corresponds to the case of $\tan\beta \gg 1$, in Fig.2(a), where the combined experimental upper bound of the neutron EDM[16], $8 \times 10^{-26}\mathrm{e} \cdot \mathrm{cm}$, is shown by the horizontal dotted line. The two lightest Higgs scalars have been taken into account in our calculations. Defining the two lightest Higgs scalar masses to be m_{H1} and m_{H2}, we fixed tentatively $m_{H1} = 200$GeV and $m_{H2} = 250$GeV, which correspond to $h = -0.37$. In Fig.2(a), the contributions of O_{ug} and O_{3g} are shown multiplying them by the factor 100 because they are very small. It is noted that the signs of these two contributions are opposite, and they almost cancel each other. The main contribution follows from the one of $O_{dg} + O_{sg}$, in which the operator O_{sg} is dominant due to the s-quark mass. This contribution is constant versus $\tan\beta$ since the $\tan\beta$ dependence of $C_{dg} + C_{sg}$ disappears as seen in the Eqs.(2) and (23), and then overlapps perfectly to the total EDM(solid line) in Fig.2(a). Thus, the O_{sg} operator dominates the neutron EDM in the case of $\tan\beta \gg 1$.

Fig.2(a)

As the mass difference of these two Higgs scalar masses becomes smaller, the neutron EDM is considerably reduced since the second Higgs scalar exchange contributes in the opposite sign to the lightest Higgs scalar one. In Fig.2(b), we show the the predicted neutron EDM versus m_{H1}/m_{H2} in the case of $\tan\beta = 10$ with $m_{H1} = 200$ and 400GeV. As far as $m_{H1}/m_{H2} \geq 0.7(|h| \leq 0.68)$, the predicted value lies under the experimental upper bound. Thus, it is found that the second lightest Higgs scalar also significantly contributes to CP violation.

Fig.2(b)

The neutron EDM in the case of $\tan\beta \ll 1$ is shown in Fig.3(a), where we take the region of $\tan\beta \leq 0.25$. The contributions of O_{ug} and O_{3g} become very large due to the large ImZ_2. However, these contribute to the neutron EDM in opposite signs, so they almost cancel each other in the region of $1 \gg \tan\beta \geq 0.1$ as shown in Fig.3(a). The remaining contribution is the one of $O_{dg} + O_{sg}$, which is constant versus $\tan\beta$. In the region of $\tan\beta \leq 0.1$, the cancelation between O_{ug} and O_{3g} is violated and the contribution of O_{ug} donimates the neutron EDM in the region of $\tan\beta \ll 0.1$.

Fig.3(a)

In Fig.3(b), we show the predicted neutron EDM versus m_{H1}/m_{H2} in the case of $\tan\beta = 0.1$. The allowed parameter region of m_{H1}/m_{H2} is obtained by the experiment and is $m_{H1}/m_{H2} \geq 0.95(|h| \leq 0.07)$. In othe words, the second lightest Higgs scalar mass should be close to the lightest one. We want to note that the predicted EDM with $m_{H1} = 400$GeV is larger than the one with $m_{H1} = 200$GeV in the region of $m_{H1}/m_{H2} \geq 0.5$. The cancelation between O_{ug} and O_{3g} is violated and the contribution of O_{3g} dominates the neutron EDM in the case of $m_{H1} = 400$GeV at $\tan\beta = 0.1$. Thus, one should carefully analyze the signs and magnitudes of the contribution of O_{ug}, $O_{dg} + O_{sg}$ and O_{3g} operators in the case of $\tan\beta \ll 1$ since those sensitively depend on the values of m_{H1}, m_{H2} and $\tan\beta$.

Fig.3(b)

The neutron EDM in the case of $\tan\beta \simeq 1$ is shown in Fig.4(a). Since the parameter h is independent of the Higgs scalar mass difference in contrast to the above two cases, we fix $h = -0.1$ as a typical value with $m_{H1} = 200$GeV and $m_{H2} = 250$GeV. The contributions of O_{ug} and O_{3g} are shown multiplying them by the factor 10. Similarly to be former cases, the signs of these two contributions are opposite and cancel each other, and so the dominant contribution is the one of $O_{dg} + O_{sg}$, which overlapps perfectly to the total EDM(solid line) in Fig.4(a).

Fig.4(a)

In Fig.4(b), the predicted neutron EDM is shown versus m_{H1}/m_{H2} in the case of $\tan\beta = 1$ with $h = -0.05, -0.1$. In the region of $m_{H1}/m_{H2} = 0.5 \sim 0.9$, the predicted value is over the experimental upper bound in the case of $m_{H1} = 200$GeV with $h = -0.1$. Thus, the magnitude of $|h|$ is rigorously restricted by the experimental upper bound of the neutron EDM. In both regions of the large and small m_{H1}/m_{H2}, the predicted neutron EDM is reduced. At $m_{H1}/m_{H2} \simeq 1$, the cancellation mechanism by the second lightest Higgs scalar operates well, while around $m_{H1}/m_{H2} \simeq 0$, the large mass difference of the two Higgs scalars leads to the small θ_{23} as seen in Eq.(20).

Fig.4(b)

In all cases of $\tan\beta$, the contribution of $O_{dg} + O_{sg}$ dominate the neutron EDM. The effects of O_{ug} and O_{3g} seem to become large only in the region of $\tan\beta \ll 1$ although these cancel each other considerably.

5. Conclusion

We have studied the effects of the four operators O_{ug}, $O_{dg} + O_{sg}$ and O_{3g} on the neutron EDM. The contribution of O_{sg} dominates over that of other operators except for the region of $\tan\beta \ll 1$. Moreover, the contributions of O_{ug} and O_{3g} cancel out each other due their opposite signs. This qualitative situation does not depend on the detail of the strong interaction hadronic model. Actually, in the NDA approximation[7] of the hadronic effect, the effects of the two operators almost cancel out although the predicted EDM is smaller than ours by a factor $2 \sim 3$. Thus, the Weinberg's three gluon operator is not a main source of the neutron EDM in THDM. Of course, Weinberg's operator may be dominant one in the other models beyond SM, which we will investigate elsewhere. The CP violation mainly follows from the two light neutral Higgs scalar exchanges. Since these two exchange contributions are of opposite signs, the CP violation is considerably reduced if the mass difference of the two Higgs scalars is within the order of $O(50\mathrm{GeV})$.

Since our results have been shown by taking $\sin\phi = \pi/4$, for an arbitrary ϕ our predicted neutron EDM is simply scaled by the factor $\sin 2\phi$. This factor is expected to be of the order one unless ϕ is suppressed by an unknown mechanism in THDM. Therefore, our results remain unchanged qualitatively.

Since our predicted neutron EDM lies around the present experimental bound, its experimental improvement may reveal the new physics beyond SM.

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Figure Captions

Fig.1: The predicted CP violation factors in the case of $\phi = \pi/4$. The solid curves show (a)ImZ_1 and (b)ImZ_2 in $\tan\beta = 5 \sim 10$, (c)ImZ_1 and (d)ImZ_2 in $\tan\beta = 0 \sim 0.3$, (e)ImZ_1 and (f)ImZ_2 in $\tan\beta = 0.8 \sim 1.2$ with $M_2 = 200$GeV, $M_3 = 250$GeV and $h = -0.1$. The dashed curves denote the upper bounds given by Weinberg.

Fig.2(a): The predicted neutron EDM in $\tan\beta = 5 \sim 10$ with $m_{H1} = 200$GeV and $m_{H2} = 250$GeV. The dotted curve and dashed curve denote the contribution by O_{ug} and O_{3g}, respectively. The contribution of $O_{dg} + O_{sg}$ overlapps the total neutron EDM shown by the solid line. The horizontal dotted line denotes the experimental upper bound.

Fig.2(b): The m_{H1}/m_{H2} dependence of the neutron EDM in $\tan\beta = 10$ with $m_{H1} = 200, 400$GeV.

Fig.3(a): The predicted neutron EDM in $\tan\beta = 0 \sim 0.3$ with $m_{H1} = 200$GeV and $m_{H2} = 250$GeV. The notations are same as in Fig.2(a). The dashed horizontal line denotes the contribution by $O_{dg} + O_{sg}$.

Fig.3(b): The m_{H1}/m_{H2} dependence of the neutron EDM in $\tan\beta = 0.1$ with $m_{H1} = 200, 400$GeV.

Fig.4(a): The predicted neutron EDM in $\tan\beta = 0.8 \sim 1.2$ with $m_{H1} = 200$GeV and $m_{H2} = 250$GeV. The notations are same as in Fig.2(a).

Fig.4(b): The m_{H1}/m_{H2} dependence of the neutron EDM in $\tan\beta = 1$ with $m_{H1} = 200, 400$GeV and $h = -0.05, -0.1$.