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Non-Anomalous Flavor Symmetries and $SU(6) \times SU(2)_R$ Model

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Abstract

We introduce the flavor symmetry $\mathbf{Z}_M \times \mathbf{Z}_N \times D_4$ into the $SU(6) \times SU(2)_{\rm R}$ string-inspired model. The cyclic group $\mathbf{Z}_M$ and the dihedral group D_4 are R symmetries, while $\mathbf{Z}_N$ is a non-R symmetry. By imposing the anomaly-free conditions on the model, we obtain a viable solution under many phenomenological constraints coming from the particle spectra. For the neutrino sector, we find a LMA-MSW solution but no SMA-MSW solution. The solution includes phenomenologically acceptable results concerning fermion masses and mixings and also concerning hierarchical energy scales including the GUT scale, the μ scale and the Majorana mass scale of R-handed neutrinos.

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1 Introduction

It is likely that in the framework of a unified theory, the characteristic patterns of fermion masses and mixings are closely linked to the flavor symmetry. In addition, it is feasible that the flavor symmetry also controls the GUT scale, the μ scale and the Majorana mass scale of R-handed neutrinos. In a previous paper[1] the authors introduced the flavor symmetry $\mathbf{Z}_M \times \mathbf{Z}_N \times D_4$ into the $SU(6) \times SU(2)_{\mathrm{R}}$ string-inspired model, where $\mathbf{Z}_M$ and $\mathbf{Z}_N$ are R and ordinary symmetries, respectively. The dihedral group D_4 is also an R symmetry. The inclusion of D_4 is motivated by the phenomenological observation that the R-handed Majorana neutrino mass for the third generation is nearly equal to the geometrical average of the string scale M_S and the electroweak scale M_Z. In the string theory it can be expected that the discrete flavor symmetries including the dihedral group D_4 arise from the symmetric structure of the compact space.

It has been pointed out that all non-gauge symmetries are strongly violated by quantum gravity effects around the Planck scale and hence in the low-energy effective theory we cannot have any global symmetries.[2] This statement holds even for the discrete symmetry introduced above. In contrast to the situation for non-gauge symmetries, if the flavor symmetries are unbroken discrete subgroups of local gauge symmteries, the discrete flavor symmetries are stable with respect to quantum gravity effects and therefore remain in the low-energy effective theory. Such discrete flavor symmetries are subject to certain anomaly cancellation conditions.[3, 4] These conditions are so stringent that many candidates of discrete symmetries are ruled out. Although in Ref.[1] the authors found interesting solutions that yield not only fermion mass hierarchies but also hierarchical energy scales, the flavor symmetry adopted there is inconsistent with the anomaly-free conditions. The purpose of this paper is to explore the non-anomalous flavor symmetry $\mathbf{Z}_M \times \mathbf{Z}_N \times D_4$ and to find phenomenologically viable anomaly free solutions.

This paper is organized as follows. In section 2 we briefly explain the main features of the $SU(6) \times SU(2)_{\mathrm{R}}$ string-inspired model, in which $\mathbf{Z}_M \times \mathbf{Z}_N$ and the dihedral group D_4 symmetries are introduced as the flavor symmetry. We use a projective representation of D_4, which is expected to arise in the theory on a compact space with non-commutative geometry. It is pointed out that the D_4 symmetry is an extension of the R-parity. In section 3 we study phenomenological constraints on the flavor charges of the matter fields. These constraints come from fermion mass hierachies and mixings and also from hierarchical energy scales. The anomaly-free conditions are given in section 4. Important conditions arise from the flavor-gauge mixed anomalies. In section 5 we solve the anomaly-free conditions, taking account of the phenomenological constraints and present a large mixing angle (LMA)-MSW solution. However, small mixing angle (SMA)-MSW solutions could not be found in the region of plausible parameter values. The distinction between these solutions results from the difference in the flavor charge assignments. We obtain phenomenologically viable results regarding fermion masses and mixings and also regarding hierarchical energy scales, including the GUT scale, the μ scale and the Majorana mass scale of R-handed neutrinos. The final section is devoted to summary and discussion.

2 $SU(6) \times SU(2)_{\rm R}$ Model

The $SU(6) \times SU(2)_{\rm R}$ string-inspired model considered here is studied in detail in Refs. [5, 6, 7, 8, 9]. In this section we review the main features of the model.

(i). The unified gauge symmetry G at the string scale M_S is assumed to be $SU(6) \times SU(2)_{\rm R}$.

(ii). Matter consists of chiral superfields of three families and one vector-like multiplet, i.e.,

$$3 \times \mathbf{27}(\Phi_{1,2,3}) + (\mathbf{27}(\Phi_0) + \overline{\mathbf{27}}(\overline{\Phi})), \tag{1}$$

in terms of E_6. The superfields Φ in $\mathbf{27}$ of E_6 are decomposed into irreducible representations of $G = SU(6) \times SU(2)_{\rm R}$ as

$$\Phi(\mathbf{27}) = \begin{cases} \phi(\mathbf{15}, \mathbf{1}) & : \quad Q, L, g, g^c, S, \\ \psi(\mathbf{6}^*, \mathbf{2}) & : \quad (U^c, D^c), (N^c, E^c), (H_u, H_d), \end{cases} \tag{2}$$

where the pair g and g^c and the pair H_u and H_d represent colored Higgs and doublet Higgs superfields, respectively, N^c is the right-handed neutrino superfield, and S is an $SO(10)$ singlet.

(iii). Gauge invariant trilinear couplings in the superpotential W take the forms

$$(\phi(\mathbf{15},\mathbf{1}))^3 = QQg + Qg^cL + g^cgS, \tag{3}$$

$$\begin{aligned}\phi(\mathbf{15},\mathbf{1})(\psi(\mathbf{6}^*,\mathbf{2}))^2 &= QH_dD^c + QH_uU^c + LH_dE^c + LH_uN^c \\ &\quad + SH_uH_d + gN^cD^c + gE^cU^c + g^cU^cD^c.\end{aligned} \tag{4}$$

The gauge group $G = SU(6) \times SU(2)_{\rm R}$ can be obtained from E_6 through the Hosotani mechanism or flux breaking on multiply-connected manifolds.[10, 11, 12] We construct the multiply-connected manifold K as the coset K_0/G_d of a simply-connected K_0 modded out by a discrete group G_d of K_0. In the presence of a background gauge field for extra-dimensional components, we have a nontrivial holonomy U_d on $K = K_0/G_d$. This nontrivial U_d gives rise to the discrete symmetry $\overline{G}_d$, which is an embedding of G_d into E_6. The unbroken gauge group G is the subgroup of E_6 whose elements commute with all elements of $\overline{G}_d$. When the holonomy U_d is of the form

$$U_d = \exp(\pi i I_3(SU(2))), \tag{5}$$

we obtain $\overline{G}_d \equiv \mathbf{Z}_2^{(\rm W)}$, where I_3 represents the third direction of an appropriate $SU(2)$ in E_6. The gauge group G becomes $SU(6) \times SU(2)$.[13] The superfield $\mathbf{27}$ of E_6 is decomposed into two irreducible representations $\phi(\mathbf{15},\ \mathbf{1})$ and $\psi(\mathbf{6}^*,\ \mathbf{2})$, which are even and odd under $\mathbf{Z}_2^{(\rm W)}$ parity, respectively.

In the conventional GUT-type models, unless an adjoint or higher representation matter (Higgs) field develops a non-zero VEV, it is impossible for a large gauge symmetry such as $SU(5)$ or $SO(10)$ to be spontaneously broken down to the standard model gauge group $G_{\rm SM}$ via the Higgs mechanism. Contrastingly, in the present model, matter fields consist only of $\mathbf{27}$ and $\overline{\mathbf{27}}$. The symmetry breaking of $G = SU(6) \times SU(2)_{\rm R}$ down to $G_{\rm SM}$ can take place via the Higgs mechanism without matter fields of adjoint or higher representations. In addition, $SU(6) \times SU(2)_{\rm R}$ is the largest of such gauge groups. Furthermore, it should be noted that doublet Higgs and color-triplet Higgs fields belong to different irreducible representations of G, as shown in Eq. (2). As a consequence, the triplet-doublet splitting problem is solved naturally.[5]

As the flavor symmetry, we introduce $\mathbf{Z}_M \times \mathbf{Z}_N$ and D_4 symmetries and regard $\mathbf{Z}_M$ and $\mathbf{Z}_N$ as the R and non-R symmetries, respectively. Assuming that M and N are relatively prime, we combine these symmetries as

$$\mathbf{Z}_M \times \mathbf{Z}_N = \mathbf{Z}_{MN}. \tag{6}$$

Table 1: Assignment of $\mathbf{Z}_{MN}$ charges for matter superfields

	$\Phi_i\ (i=1,2,3)$	Φ_0	$\overline{\Phi}$
$\phi(\mathbf{15},\ \mathbf{1})$	a_i	a_0	$\overline{a}$
$\psi(\mathbf{6}^*,\ \mathbf{2})$	b_i	b_0	$\overline{b}$

In this case we stipulate that the Grassmann number θ in the superfield formalism has the charge $(\pm 1,\ 0)$ under $\mathbf{Z}_M \times \mathbf{Z}_N$. The charge of θ under $\mathbf{Z}_{MN}$ is denoted as q_θ, which becomes a multiple of N, and $q_\theta \equiv \pm 1 \pmod{M}$. The $\mathbf{Z}_{MN}$ charges of matter superfields are denoted as a_i and b_i, etc., as shown in Table I.

Introduction of the dihedral group $D_4 = \mathbf{Z}_2 \ltimes \mathbf{Z}_4$ is motivated by the phenomenological observation that the R-handed Majorana neutrino mass for the third generation is nearly equal to the geometrical average of M_S and M_Z. The $\mathbf{Z}_2$ and $\mathbf{Z}_4$ groups are expressed as

$$\mathbf{Z}_2 = \{1,\ g_1\}, \qquad \mathbf{Z}_4 = \{1,\ g_2,\ g_2^2,\ g_2^3\}, \tag{7}$$

respectively, and we have the relation

$$g_1 g_2 g_1^{-1} = g_2^{-1}. \tag{8}$$

The elements g_1 and g_2 correspond to reflection and rotation by $\pi/2$ of a square, respectively.

The reader might think that the D_4 symmetry is somewhat unfamiliar as the flavor symmetry. However, examples of D_4 symmetric Calabi-Yau space can be easily constructed as follows. We first note that zero locus of the 5th-order defining polynomial in CP^4 is a simple example of the Calabi-Yau space. Denoting the homogeneous coordinates of CP^4 as z_i $(i = 1,\ 2,\ \cdots 5)$, we take the defining polynomial as

$$P(z) = \sum_{i=1}^{5} z_i^5 + c z_5^3 (z_1 z_3 + z_2 z_4), \tag{9}$$

where c is a complex constant. The defining polynomial $P(z)$ is invariant under the transformation

$$g_1\ :\ z_1 \leftrightarrow z_3, \quad z_i \to z_i, \quad (i = 2, 4, 5) \tag{10}$$

which composes $\mathbf{Z}_2^{(\mathrm{A})} = \{1,\ g_1\}$, and also under the transformation

$$g_2\ :\ z_1 \longrightarrow z_2 \longrightarrow z_3 \longrightarrow z_4 \longrightarrow z_1, \quad z_5 \longrightarrow z_5, \tag{11}$$

which composes $\mathbf{Z}_4 = \{1,\ g_2,\ g_2^2,\ g_2^3\}$. These transformations yield the dihedral group $D_4 = \mathbf{Z}_2^{(\mathrm{A})} \ltimes \mathbf{Z}_4$. Then, D_4 symmetry arises on the compact space constructed here. This simple example suggests that it is not so unusual that the dihedral group D_4 is included among the flavor symmetries in the effective theory from the string compactification.

Furthermore, when c is real, instead of the above $\mathbf{Z}_2^{(\mathrm{A})}$ transformation, we may adopt another $\mathbf{Z}_2$ transformation,

$$g_1'\ :\ z_1 \longrightarrow \overline{z_3}, \quad z_3 \longrightarrow \overline{z_1}, \quad z_i \longrightarrow \overline{z_i}, \quad (i = 2, 4, 5) \tag{12}$$

which is a combined transformation $\mathbf{Z}_2^{(\mathrm{AC})}$ consisting of $\mathbf{Z}_2^{(\mathrm{A})}$ and complex conjugation. The operation of complex conjugation corresponds to the reversal of the string orientation. Under this transformation, $P(z)$ transforms into $P(\overline{z}) = \overline{P(z)}$. Then, the defining polynomial remains essentially unchanged. Although chiral matter superfields transform into anti-chiral ones, the terms coming from the superpotential

$$\int d\theta^2\, W + \int d\overline{\theta}^2\, \overline{W} \tag{13}$$

are invariant under the $\mathbf{Z}_2^{(\mathrm{AC})}$ transformation, provided that θ ($\overline{\theta}$) transforms into $\overline{\theta}$ (θ) simultaneously.

It is assumed that the flavor symmetry contains the dihedral group D_4. Here we denote this D_4 as $\mathbf{Z}_2^{(\mathrm{F})} \ltimes \mathbf{Z}_4$. In a string with discrete torsion, the coordinates in the compact space become non-commutative and are represented by a projective representation of the flavor symmetry.[14, 15, 16] This non-commutativity of the coordinates corresponds to brane fluctuations. The non-commutative coordinates are concretely represented in terms of matrices.[17] Massless matter fields in the effective theory correspond to the degree of freedom of deformation of the compact space and are expressed by functions of non-commutative coordinates. Therefore, massless matter fields turn out to be of matrix form. Specifically, the matter fields are described in terms of the ordinary four-dimensional fields mutiplied by the matrices associated with the non-commutativity of the compact space. The four-dimensional Lagrangian of the theory should belong to the center of the non-commutative algebra.

As pointed out in Ref.[1], this implies that a new type of flavor symmetry arises in the theory on a compact space with non-commutative geometry.

To begin with, let us consider a projective representation of the dihedral group $D_4 = \mathbf{Z}_2^{(\mathrm{F})} \ltimes \mathbf{Z}_4$. It is easy to see that a projective representation of this D_4 is given by the unitary matrices

$$\gamma(g_1) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_1, \qquad \gamma(g_2) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \equiv \sigma_4, \tag{14}$$

which satisfy the relations

$$\gamma(g_1)\,\gamma(g_2)\,\gamma(g_1)^{-1} = i\,\gamma(g_2)^{-1}, \qquad \gamma(g_1)^2 = \gamma(g_2)^4 = 1. \tag{15}$$

In this case we have

$$\gamma(g_1\, g_2^2) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_2, \qquad \gamma(g_2^2) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_3. \tag{16}$$

In D_4 there exist five conjugacy classes,

$$\{1\}, \quad \{g_1,\ g_1 g_2^2\}, \quad \{g_2^2\}, \quad \{g_2,\ g_2^3\}, \quad \{g_1 g_2,\ g_1 g_2^3\}. \tag{17}$$

Correspondingly, for example, 1 and σ_i $(i = 1,2,3,4)$ transform as

$$\gamma(g_1)\,\{1,\ \sigma_1,\ \sigma_2,\ \sigma_3,\ \sigma_4\}\,\gamma(g_1)^{-1} = \{1,\ \sigma_1,\ -\sigma_2,\ -\sigma_3,\ i\sigma_4^{-1}\}, \tag{18}$$
$$\gamma(g_2)\,\{1,\ \sigma_1,\ \sigma_2,\ \sigma_3,\ \sigma_4\}\,\gamma(g_2)^{-1} = \{1,\ \sigma_2,\ -\sigma_1,\ \sigma_3,\ \sigma_4\}. \tag{19}$$

To each matter superfield we assign a "D_4-charge" which is expressed in terms of the representation matrices of D_4.

We now define a combined transformation $\mathbf{Z}_2^{(\mathrm{FC})}$ consisting of $\mathbf{Z}_2^{(\mathrm{F})}$ and hermitian conjugation. In addition, we define a combined transformation consisting of $\mathbf{Z}_2^{(\mathrm{FC})}$ and $\mathbf{Z}_2^{(\mathrm{W})}$ by $\mathbf{Z}_2^{(\mathrm{FCW})}$ and require that the theory be $\mathbf{Z}_2^{(\mathrm{FCW})}$ gauge-invariant. This means that the theory on a manifold K_0 is modded out by the combined $\mathbf{Z}_2^{(\mathrm{FCW})}$. Because the field $\phi(\mathbf{15},\ \mathbf{1})$ is even under $\mathbf{Z}_2^{(\mathrm{W})}$, $\mathbf{Z}_2^{(\mathrm{FC})}$ odd states are projected out for $\phi(\mathbf{15},\ \mathbf{1})$. Then the representation of D_4 for the field $\phi(\mathbf{15},\ \mathbf{1})$ is 1 or σ_1. This situation is described in Table II. In this table, σ_4 is redefined by attaching the phase

Table 2: $\mathbf{Z}_2$ parities of matter superfields

	$\mathbf{Z}_2^{(W)}$	$\mathbf{Z}_2^{(FC)}$	$\mathbf{Z}_2^{(FCW)}$
$(\mathbf{15},\ \mathbf{1})\ 1$	$+$	$+$	$+$
$(\mathbf{15},\ \mathbf{1})\ \sigma_1$	$+$	$+$	$+$
$(\mathbf{15},\ \mathbf{1})\ \sigma_2$	$+$	$-$	$-$
$(\mathbf{15},\ \mathbf{1})\ \sigma_3$	$+$	$-$	$-$
$(\mathbf{15},\ \mathbf{1})\ \sigma_4$	$+$	$-$	$-$
$(\mathbf{15},\ \mathbf{1})\ \sigma_4^{-1}$	$+$	$-$	$-$
$(\mathbf{6}^*,\ \mathbf{2})\ 1$	$-$	$+$	$-$
$(\mathbf{6}^*,\ \mathbf{2})\ \sigma_1$	$-$	$+$	$-$
$(\mathbf{6}^*,\ \mathbf{2})\ \sigma_2$	$-$	$-$	$+$
$(\mathbf{6}^*,\ \mathbf{2})\ \sigma_3$	$-$	$-$	$+$
$(\mathbf{6}^*,\ \mathbf{2})\ \sigma_4$	$-$	$-$	$+$
$(\mathbf{6}^*,\ \mathbf{2})\ \sigma_4^{-1}$	$-$	$-$	$+$

Table 3: Assignment of "D_4 charges" to matter superfields

	$\Phi_i\ (i=1,2,3)$	Φ_0	$\overline{\Phi}$
$\phi(\mathbf{15},\ \mathbf{1})$	σ_1	1	1
$\psi(\mathbf{6}^*,\ \mathbf{2})$	σ_2	σ_3	σ_4

factor $\exp(i\pi/4)$. Then, σ_4 and σ_4^{-1} become odd under $\mathbf{Z}_2^{(FC)}$. On the other hand, since $\psi(\mathbf{6}^*,\ \mathbf{2})$ is odd under $\mathbf{Z}_2^{(W)}$, $\mathbf{Z}_2^{(FC)}$ even states are projected out for $\psi(\mathbf{6}^*,\ \mathbf{2})$. Then the representation of D_4, i.e. σ_2, σ_3, σ_4 or σ_4^{-1}, is attached to the field $\psi(\mathbf{6}^*,\ \mathbf{2})$. The disappearance of σ_1 (σ_2) from the spectra of $\psi(\mathbf{6}^*,\ \mathbf{2})$ ($\phi(\mathbf{15},\ \mathbf{1})$) induces the breakdown of $D_4 = \mathbf{Z}_2^{(F)} \ltimes \mathbf{Z}_4$ to $\mathbf{Z}_2^{(F)} \times \mathbf{Z}_2$.

We are now in a position to assign the "D_4-charges" to matter fields, as shown in Table III, and σ_1 to the Grassmann number θ. It is worth noting that the σ_3 transformation yields the R-parity. In fact, we find that

$$\sigma_3\sigma_1\sigma_3^{-1} = -\sigma_1, \qquad \sigma_3\sigma_2\sigma_3^{-1} = -\sigma_2 \tag{20}$$

Table 4: R-parities of matter superfields

	Φ_i $(i=1,2,3)$	Φ_0	$\overline{\Phi}$
$\phi(\mathbf{15},\ \mathbf{1})$	$-$	$+$	$+$
$\psi(\mathbf{6}^*,\ \mathbf{2})$	$-$	$+$	$+$

and

$$\sigma_3 1 \sigma_3^{-1} = 1, \qquad \sigma_3\sigma_4\sigma_3^{-1} = \sigma_4, \qquad \sigma_3\sigma_3\sigma_3^{-1} = \sigma_3. \tag{21}$$

In other words, the R-parities of the superfields Φ_i $(i=1,2,3)$ for three generations are all odd, while those of Φ_0 and $\overline{\Phi}$ are even. This is shown in Table IV. Therefore, the dihedral flavor symmetry D_4 is an extension of the R-parity. When $\psi(\mathbf{6}^*,\ \mathbf{2})_0$ and $\overline{\psi(\mathbf{6}^*,\ \mathbf{2})}$ develop non-zero VEVs, the $\mathbf{Z}_2^{(\mathrm{F})}$ symmetry is spontaneously broken. Eventually, the dihedral flavor symmetry $D_4 = \mathbf{Z}_2^{(\mathrm{F})} \ltimes \mathbf{Z}_4$ is spontaneously broken down to $\mathbf{Z}_2^{(\mathrm{R})}$ symmetry. This $\mathbf{Z}_2^{(\mathrm{R})}$ symmetry is a remnant of the $\mathbf{Z}_4$ symmetry and is identified with the R-parity.

3 Fermion mass hierarchies and mixings

In this section we study phenomenological requirements, which yield many constraints on the assignments of the discrete flavor charges. Our purpose is to explain not only the fermion mass hierarchies and the mixings but also the hierachical energy scales, including the breaking scale of the GUT-type gauge symmetry, the intermediate Majorana masses of the R-handed neutrinos and the scale of the μ term.

In the R-parity even sector, it is assumed that the superpotential contains the terms

$$W_1 \sim M_S^3 \left[\lambda_0 \left(\frac{\phi_0\overline{\phi}}{M_S^2} \right)^{2n} + \lambda_1 \left(\frac{\phi_0\overline{\phi}}{M_S^2} \right)^{n} \left(\frac{\psi_0\overline{\psi}}{M_S^2} \right)^{m} + \lambda_2 \left(\frac{\psi_0\overline{\psi}}{M_S^2} \right)^{2m} \right], \tag{22}$$

with $\lambda_i = \mathcal{O}(1)$, where the exponents are non-negative integers that satisfy the $\mathbf{Z}_{MN}$

symmetry conditions,

$$\begin{aligned} 2n(a_0+\overline{a})-2q_\theta &\equiv 0, \\ n(a_0+\overline{a})+m(b_0+\overline{b})-2q_\theta &\equiv 0, \qquad (\mathrm{mod}\ MN) \\ 2m(b_0+\overline{b})-2q_\theta &\equiv 0. \end{aligned} \tag{23}$$

The dihedral symmetry D_4 requires $m \equiv 0$ (mod 4). Then, for the sake of simplicity we put $m = 4$. As discussed in Ref.[1], we consider the case that M is odd and $N \equiv 2$ (mod 4). Furthermore, the $\mathbf{Z}_{MN}$ charges are chosen as

$$a_0+\overline{a}=-4, \qquad \overline{b}=\mathrm{odd}, \qquad a_i,\ b_i=\mathrm{even},\ (i=0,1,2,3) \tag{24}$$

and

$$a_i+a_j,\ a_0,\ \overline{a},\ b_i+b_j \equiv 0. \qquad (\mathrm{mod}\ 4) \tag{25}$$

In this case we obtain

$$n=\frac{1}{4}(MN-q_\theta)=-(b_0+\overline{b}). \tag{26}$$

Through the minimization of the scalar potential with the soft SUSY breaking mass terms characterized by the scale $\widetilde{m}_0 \sim 10^3$ GeV, matter fields develop non-zero VEVs. In Refs. [18] and [19] we studied the minimum point of the scalar potential in detail. The gauge symmetry is spontaneously broken in two steps with a feasible parameter region of the coefficients λ_i. The scales of the gauge symmetry breaking are given by

$$\begin{aligned} |\langle\phi_0\rangle|=|\langle\overline{\phi}\rangle| &= M_S\,\rho^{1/2(2n-1)}, \\ |\langle\psi_0\rangle|=|\langle\overline{\psi}\rangle| &\simeq M_S\,\rho^{n/8(2n-1)}. \end{aligned} \tag{27}$$

The parameter ρ is defined by $\rho = c\widetilde{m}_0/M_S$, where $c \simeq n^{-3/2} f(\lambda_0, \lambda_1, \lambda_2)$. Here we take the numerical values $M_S \sim 5\times10^{17}$GeV[20] and $c \sim 10^{-2}$. Thus, we have $\rho \sim 2\times10^{-17}$. The D-flat conditions require $|\langle\phi_0\rangle| = |\langle\overline{\phi}\rangle|$ and $|\langle\psi_0\rangle| = |\langle\overline{\psi}\rangle|$ with $\mathcal{O}(M_S\,\rho)$ accuracy. Under the assumption $n > m = 4$, we have

$$|\langle\phi_0\rangle| > |\langle\psi_0\rangle|. \tag{28}$$

In what follows, we use the notation

$$\frac{\langle\phi_0\rangle\langle\overline{\phi}\rangle}{M_S^2}=x, \qquad \frac{\langle\psi_0\rangle\langle\overline{\psi}\rangle}{M_S^2}=x^{\frac{n}{4}+\delta_N}, \tag{29}$$

with $x^{\delta_N} \sim 1$. Then we have

$$x^{2n-1} = \rho \sim 2 \times 10^{-17}. \tag{30}$$

The gauge symmetry is spontaneously broken at the scale $|\langle \phi_0(\mathbf{15},\ \mathbf{1}) \rangle|$, and subsequently at the scale $|\langle \psi_0(\mathbf{6}^*,\ \mathbf{2}) \rangle|$. This yields the symmetry breakings

$$SU(6) \times SU(2)_{\rm R} \xrightarrow{\langle \phi_0 \rangle} SU(4)_{\rm PS} \times SU(2)_L \times SU(2)_{\rm R} \xrightarrow{\langle \psi_0 \rangle} G_{\rm SM}, \tag{31}$$

where $SU(4)_{\rm PS}$ is the Pati-Salam $SU(4)$.[21] Since the fields that develop non-zero VEVs are singlets under the remaining gauge symmetries, they are assigned as $\langle \phi_0(\mathbf{15},\ \mathbf{1}) \rangle = \langle S_0 \rangle$ and $\langle \psi_0(\mathbf{6}^*,\ \mathbf{2}) \rangle = \langle N_0^c \rangle$. Below the scale $|\langle \phi_0 \rangle|$, the Froggatt-Nielsen mechanism acts for non-renormalizable interactions.[22] In the first step of the symmetry breaking, the fields Q_0, L_0, $\overline{Q}$, $\overline{L}$ and $(S_0 - \overline{S})/\sqrt{2}$ are absorbed by the gauge fields. Through subsequent symmetry breaking, the fields U_0^c, E_0^c, $\overline{U}^c$, $\overline{E}^c$ and $(N_0^c - \overline{N}^c)/\sqrt{2}$ are absorbed.

The colored Higgs mass arises from the term

$$z_{00} \left(\frac{S_0 \overline{S}}{M_S^2} \right)^{\zeta_{00}} S_0 g_0 g_0^c, \tag{32}$$

with $z_{00} = \mathcal{O}(1)$. The $\mathbf{Z}_{MN}$ symmetry controls the exponent ζ_{00} as

$$-4\zeta_{00} + 3a_0 - 2q_\theta \equiv 0, \qquad (\mathrm{mod}\ MN) \tag{33}$$

where we have used $a_0 + \overline{a} = -4$. Due to the Froggatt-Nielsen mechanism, the colored Higgs mass can be expressed as

$$m_{g_0/g_0^c} \sim x^{\zeta_{00}} \langle S_0 \rangle. \tag{34}$$

In order to guarantee the longevity of the proton, ζ_{00} should be sufficiently small compared to n. For this reason, we rewrite Eq. (33) as

$$-4\zeta_{00} + 3a_0 - 2q_\theta = 0, \tag{35}$$

which gives a small non-negative value of ζ_{00} when $3a_0 - 2q_\theta$ is a small non-negative multiple of 4.

Similarly, the μ term induced from

$$h_{00}\left(\frac{S_0\overline{S}}{M_S^2}\right)^{\eta_{00}} S_0 H_{u0} H_{d0}, \tag{36}$$

with $h_{00}=\mathcal{O}(1)$, is of the form

$$\mu \sim x^{\eta_{00}} \langle S_0 \rangle. \tag{37}$$

The exponent η_{00} is determined by

$$-4\eta_{00}+a_0+2b_0-2q_\theta \equiv 0. \qquad (\mathrm{mod}\ MN) \tag{38}$$

In order to obtain $\mu \sim \mathcal{O}(10^2)$GeV, we need $\eta_{00} \sim 2n$. Then, when b_0 is even and $a_0+2b_0 \leq 0$, we rewrite Eq. (38) as

$$-4\eta_{00}+a_0+2b_0-2q_\theta = -2MN. \tag{39}$$

We now turn to the quark/lepton mass matrices. The mass matrix for up-type quarks comes from the term

$$m_{ij}\left(\frac{S_0\overline{S}}{M_S^2}\right)^{\mu_{ij}} Q_i U_j^c H_{u0}, \qquad (i,\ j=1,\,2,\,3) \tag{40}$$

with $m_{ij}=\mathcal{O}(1)$. The exponent μ_{ij} is determined by

$$-4\mu_{ij}+a_i+b_j+b_0-2q_\theta \equiv 0. \qquad (\mathrm{mod}\ MN) \tag{41}$$

The 3×3 mass matrix is given by

$$\mathcal{M}_{ij} v_u = m_{ij}\, x^{\mu_{ij}} v_u\,, \tag{42}$$

where $v_u=\langle H_{u0}\rangle$. In order to account for the experimental fact that the top-quark mass is of $\mathcal{O}(v_u)$, we expect $\mu_{33}\simeq 0$ and set

$$-4\mu_{33}+a_3+b_3+b_0-2q_\theta = 0. \tag{43}$$

This relation holds when $a_3+b_3+b_0 \equiv 0$ (mod 4). Furthermore, we choose the parameterization

$$\alpha_1>\alpha_2>\alpha_3=0, \qquad \beta_1>\beta_2>\beta_3=0, \tag{44}$$

where α_i and β_i are integers defined by

$$\alpha_i = \frac{1}{4}(a_i - a_3), \qquad \beta_i = \frac{1}{4}(b_i - b_3). \tag{45}$$

Then the mass matrix Eq. (42) is of the form

$$\mathcal{M} \times v_u = \begin{pmatrix} m_{11}x^{\alpha_1+\beta_1} & m_{12}x^{\alpha_1+\beta_2} & m_{13}x^{\alpha_1} \\ m_{21}x^{\alpha_2+\beta_1} & m_{22}x^{\alpha_2+\beta_2} & m_{23}x^{\alpha_2} \\ m_{31}x^{\beta_1} & m_{32}x^{\beta_2} & m_{33} \end{pmatrix} \times x^{\mu_{33}}\, v_u. \tag{46}$$

The mass eigenvalues for up-type quarks become

$$(m_u,\ m_c,\ m_t) \sim (x^{\alpha_1+\beta_1},\ x^{\alpha_2+\beta_2},\ 1) \times x^{\mu_{33}}\, v_u \tag{47}$$

at the string scale M_S.

In the down-quark sector, the mass matrix is given by[5, 6, 7, 8]

$$\widehat{\mathcal{M}}_d = \begin{matrix} \\ g \\ D \end{matrix} \begin{matrix} \quad g^c \qquad D^c \\ \begin{pmatrix} y_S \mathcal{Z} & y_N \mathcal{M} \\ 0 & \rho_d \mathcal{M} \end{pmatrix} \end{matrix} \tag{48}$$

in M_S units, where $y_S = \langle S_0 \rangle / M_S$, $y_N = \langle N_0^c \rangle / M_S$, $\rho_d = v_d / M_S$ and $v_d = \langle H_{d0} \rangle$. Since g^c and D^c are indistinguishable under the standard model gauge group $G_{\rm SM}$, mixings occur between these fields. Consequently, the mass matrix for down-type quarks becomes a 6×6 matrix. The above 3×3 g-g^c submatrix coming from the term

$$z_{ij} \left(\frac{S_0 \overline{S}}{M_S^2} \right)^{\zeta_{ij}} S_0 g_i g_j^c \tag{49}$$

is given by

$$\mathcal{Z}_{ij} = z_{ij}\, x^{\zeta_{ij}}, \tag{50}$$

with $z_{ij} = \mathcal{O}(1)$. Flavor symmetry requires the conditions

$$-4\zeta_{ij} + a_i + a_j + a_0 - 2q_\theta \equiv 0. \qquad (\text{mod } MN) \tag{51}$$

Then, with $a_i + a_j \equiv 0 \pmod 4$, as shown in Eq. (25), we set

$$-4\zeta_{33} + 2a_3 + a_0 - 2q_\theta = 0. \tag{52}$$

The eigenstates of the mass matrix (48) contain three heavy modes and three light modes. An important phenomenological constraint results from the observed pattern of quark mixings. As pointed out in Ref. [8], when the relation

$$x^{\delta_d} \sim 1 \tag{53}$$

is satisfied, where

$$\delta_d = \left[\alpha_1 + \zeta_{33} + \frac{1}{2}\right] - \left[\beta_1 + \mu_{33} + \frac{1}{2}\left(\frac{n}{4} + \delta_N\right)\right], \tag{54}$$

we obtain

$$\theta_{12} \sim x^{\alpha_1 - \alpha_2}, \qquad \theta_{23} \sim x^{\alpha_2}. \tag{55}$$

By taking $x^{\alpha_1} \sim \lambda^3$ and $x^{\alpha_2} \sim \lambda^2$ with $\lambda \sim 0.22$, we can reproduce a phenomenologically acceptable pattern of the CKM matrix :

$$V_{\rm CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^5 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \tag{56}$$

At the string scale M_S, the mass spectra of the light modes become

$$(m_d,\ m_s,\ m_b) \sim (x^{\alpha_1+\beta_1+\delta_d},\ x^{\alpha_2+\beta_1},\ x^{\alpha_2+\beta_1-\alpha_1+\delta_d}) \times x^{\mu_{33}}\, v_d \tag{57}$$

for $\delta_d \geq 0$, and

$$(m_d,\ m_s,\ m_b) \sim (x^{\alpha_1+\beta_1},\ x^{\alpha_2+\beta_1+\delta_d},\ x^{\alpha_2+\beta_1-\alpha_1+\delta_d}) \times x^{\mu_{33}}\, v_d \tag{58}$$

for $\delta_d < 0$.

In the charged lepton sector, the mass matrix has the 6×6 form[5, 6, 7, 9]

$$\widehat{\mathcal{M}}_l = \begin{array}{c} \\ H_d^- \\ L^- \end{array} \!\!\begin{array}{c} \begin{array}{cc} H_u^+ & E^{c+} \end{array} \\ \begin{pmatrix} y_S\mathcal{H} & 0 \\ y_N\mathcal{M} & \rho_d\mathcal{M} \end{pmatrix} \end{array} \tag{59}$$

in M_S units. Because H_d and L also have the same quantum number under $G_{\rm SM}$, mixings occur between these fields. The above 3×3 H_d-H_u submatrix coming from

$$h_{ij}\left(\frac{S_0\overline{S}}{M_S^2}\right)^{\eta_{ij}} S_0 H_{di} H_{uj} \tag{60}$$

is expressed as

$$\mathcal{H}_{ij} = h_{ij} x^{\eta_{ij}}, \tag{61}$$

with $h_{ij}=\mathcal{O}(1)$. ¿From the flavor symmetry, we have the conditions

$$-4\eta_{ij} + b_i + b_j + a_0 - 2q_\theta \equiv 0. \qquad (\text{mod } MN) \tag{62}$$

Again, by assuming $b_i + b_j \equiv 0$ (mod 4), we set

$$-4\eta_{33} + 2b_3 + a_0 - 2q_\theta = 0. \tag{63}$$

The eigenstates of the mass matrix (59) contain three heavy modes and three light modes.

In the neutral sector, there exist five types of matter fields, H_u^0, H_d^0, L^0, N^c and S. Then we have the 15×15 mass matrix[5, 6, 7, 9]

$$\widehat{\mathcal{M}}_{NS} = \begin{array}{c} \\ H_u^0 \\ H_d^0 \\ L^0 \\ N^c \\ S \end{array} \!\!\begin{array}{c} \begin{array}{ccccc} H_u^0 & H_d^0 & L^0 & N^c & S \end{array} \\ \left(\begin{array}{ccccc} 0 & y_S\mathcal{H} & y_N\mathcal{M}^T & 0 & \rho_d\mathcal{M}^T \\ y_S\mathcal{H} & 0 & 0 & 0 & \rho_u\mathcal{M}^T \\ y_N\mathcal{M} & 0 & 0 & \rho_u\mathcal{M} & 0 \\ 0 & 0 & \rho_u\mathcal{M}^T & \mathcal{N} & \mathcal{T}^T \\ \rho_d\mathcal{M} & \rho_u\mathcal{M} & 0 & \mathcal{T} & \mathcal{S} \end{array}\right) \end{array} \tag{64}$$

in M_S units, where $\rho_u = v_u/M_S$. In this matrix, the 6×6 submatrix

$$\widehat{\mathcal{M}}_M = \begin{pmatrix} \mathcal{N} & \mathcal{T}^T \\ \mathcal{T} & \mathcal{S} \end{pmatrix} \tag{65}$$

plays the role of the Majorana mass matrix in the seesaw mechanism. The 3×3 submatrix $\mathcal{N}$ is induced from the terms

$$M_S^{-1}\left(\frac{S_0\overline{S}}{M_S^2}\right)^{\nu_{ij}} (\psi_i\overline{\psi})(\psi_j\overline{\psi}), \qquad (i,j=1,2,3) \tag{66}$$

where the exponents are given by

$$-4\nu_{ij} + b_i + b_j + 2\overline{b} - 2q_\theta \equiv 0. \qquad (\text{mod } MN) \tag{67}$$

In fact, these terms lead to the Majorana mass terms

$$M_S \, \mathcal{N}_{ij} N_i^c N_j^c \sim M_S \, x^{\nu_{ij}} \left(\frac{\langle \overline{N^c} \rangle}{M_S} \right)^2 N_i^c N_j^c. \tag{68}$$

Phenomenologically, it is desirable for the Majorana mass of the third generation to be $10^{10} - 10^{12}$ GeV. This scale is nearly equal to the geometrical average of M_S and M_Z :

$$M_S x^{\nu_{33}} \left(\frac{\langle \overline{N^c} \rangle}{M_S} \right)^2 \sim 10 \times \sqrt{M_S \, M_Z} \sim 50 \times M_S \sqrt{\rho}. \tag{69}$$

This implies

$$\nu_{33} + \frac{n}{4} \sim 0.9 \times n. \tag{70}$$

When b_3 is even but $\overline{b}$ is odd, the flavor symmetry leads to

$$-4\nu_{33} + 2b_3 + 2\overline{b} - 2q_\theta = -MN. \tag{71}$$

Because the right-hand side of this equation is not $-2MN$ but $-MN$, we can obtain solutions consistent with Eq. (70). The submatrix $\mathcal{S}$ induced from

$$M_S^{-1} \left(\frac{S_0 \overline{S}}{M_S^2} \right)^{\sigma_{ij}} (\phi_i \overline{\phi})(\phi_j \overline{\phi}) \tag{72}$$

is expressed as

$$\mathcal{S}_{ij} \sim x^{\sigma_{ij}} \left(\frac{\langle \overline{S} \rangle}{M_S} \right)^2 . \tag{73}$$

The exponents are determined by

$$-4\sigma_{ij} + a_i + a_j + 2\overline{a} - 2q_\theta \equiv 0. \qquad (\text{mod } MN) \tag{74}$$

The submatrix $\mathcal{T}$ induced from

$$M_S^{-1}\left(\frac{S_0\overline{S}}{M_S^2}\right)^{\tau_{ij}}(\phi_i\overline{\phi})(\psi_j\overline{\psi}) \tag{75}$$

is given by

$$\mathcal{T}_{ij} \sim x^{\tau_{ij}}\frac{\langle\overline{S}\rangle\langle\overline{N^c}\rangle}{M_S^2}. \tag{76}$$

The flavor symmetry yields the conditions

$$-4\tau_{ij} + a_i + b_j + \overline{a} + \overline{b} - 2q_\theta \equiv 0. \qquad (\text{mod } MN) \tag{77}$$

Because only $\overline{b}$ is taken as an odd integer, we have no solution to satisfy Eq. (77). This means that $\mathcal{T} = 0$.

We now proceed to discuss phenomenological constraints resulting from the lepton flavor mixings. As pointed out in Ref. [9], in the present framework there are two possibilities for realistic patterns of the MNS matrix, that is, the LMA-MSW solution and the SMA-MSW solution. The LMA solution can be derived when the relation

$$x^{\delta_L} \sim 1 \tag{78}$$

holds, where

$$\delta_L = \left[\frac{\alpha_1+\alpha_2}{2} + \mu_{33} + \frac{1}{2}\left(\frac{n}{4} + \delta_N\right)\right] - \left[\beta_1 + \eta_{33} + \frac{1}{2}\right]. \tag{79}$$

In this case we have

$$\begin{aligned} \tan\theta_{12}^{\rm MNS} &\sim x^{\frac{\alpha_1-\alpha_2}{2}+\delta_L} \sim \sqrt{\lambda}\,x^{\delta_L}, \\ \tan\theta_{23}^{\rm MNS} &\sim x^{\frac{\alpha_1-\alpha_2}{2}-\delta_L} \sim \sqrt{\lambda}\,x^{-\delta_L}, \\ \tan\theta_{13}^{\rm MNS} &\sim x^{\alpha_1-\alpha_2} \sim \lambda, \end{aligned} \tag{80}$$

and the mass spectra of the light charged leptons become

$$(m_e,\ m_\mu,\ m_\tau) \sim (x^{\alpha_1+\beta_1},\ x^{\beta_2+\frac{\alpha_1+\alpha_2}{2}-\delta_L},\ x^{\alpha_2}) \times x^{\mu_{33}}\,v_d. \tag{81}$$

The neutrino masses are given by

$$(m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3}) \sim (x^{2(\alpha_1-\alpha_2)},\ x^{\alpha_1-\alpha_2-2\delta_L},\ 1) \times \frac{v_u^2}{M_S}x^{2(\alpha_2+\mu_{33})-\nu_{33}-\frac{n}{4}-\delta_N}. \tag{82}$$

In a similar way, the SMA solution is obtained when the relation

$$x^{\delta_S} \sim 1 \tag{83}$$

is satisfied, where

$$\delta_S = \left[\alpha_1 + \mu_{33} + \frac{1}{2}\left(\frac{n}{4} + \delta_N\right)\right] - \left[\beta_2 + \eta_{33} + \frac{1}{2}\right]. \tag{84}$$

In this case, the parameterizations $x^{\beta_1} \sim \lambda^4$ and $x^{\beta_2} \sim \lambda^2$ lead to

$$\begin{aligned} \tan\theta_{12}^{\mathrm{MNS}} &\sim x^{\beta_1-\beta_2-\delta_S} \sim \lambda^2 x^{-\delta_S}, \\ \tan\theta_{23}^{\mathrm{MNS}} &\sim x^{\delta_S}, \\ \tan\theta_{13}^{\mathrm{MNS}} &\sim x^{\beta_1-\beta_2} \sim \lambda^2. \end{aligned} \tag{85}$$

We then obtain the mass spectra

$$(m_e,\ m_\mu,\ m_\tau) \sim (x^{\alpha_1+2\beta_1-\beta_2-\delta_S},\ x^{\alpha_1+\beta_2},\ x^{\alpha_1-\delta_S}) \times x^{\mu_{33}}\, v_d \tag{86}$$

for light charged leptons and

$$(m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3}) \sim (x^{2(\beta_1-\beta_2)},\ x^{2\delta_S},\ 1) \times \frac{v_u^2}{M_S} x^{2(\alpha_1+\mu_{33}-\delta_S)-\nu_{33}-\frac{n}{4}-\delta_N} \tag{87}$$

for neutrinos.

4 Anomaly-free conditions

It is known that all non-gauge symmetries break down around the Planck scale due to quantum gravity effects.[2] On the other hand, phenomenologically it seems that the flavor symmetries are necessary for explaining the fermion mass hierarchies and the mixings. Therefore, it would be natural for the flavor symmetries to be unbroken discrete subgroups of local gauge symmteries. If this is the case, the discrete flavor symmetries would be stable with respect to quantum gravity effects and then remains in the low-energy effective theory. Such discrete flavor symmetries should be non-anomalous.[3, 4]

If the $\mathbf{Z}_{MN}$ symmetry considered here arises from certain gauge symmetries and if anomaly cancellation does not occur via the Green-Schwartz mechanism,[23] the $\mathbf{Z}_{MN}$ symmetry itself should be non-anomalous. Because the gauge symmetry at the string scale is assumed to be $SU(6) \times SU(2)_{\rm R}$, the mixed anomaly conditions $\mathbf{Z}_{MN} \cdot (SU(6))^2$ and $\mathbf{Z}_{MN} \cdot (SU(2)_{\rm R})^2$ are imposed on the $\mathbf{Z}_{MN}$ charges of the massless matter fields. The heavy fermions decouple in $\mathbf{Z}_{MN} \cdot (SU(6))^2$ and $\mathbf{Z}_{MN} \cdot (SU(2)_{\rm R})^2$ anomalies but not in the cubic $\mathbf{Z}_{MN}^3$ and the mixed $\mathbf{Z}_{MN} \cdot (\mathrm{Graviton})^2$ anomalies. At present, however, we have no information about the heavy modes. Therefore, the cubic $\mathbf{Z}_{MN}^3$ and the mixed $\mathbf{Z}_{MN} \cdot (\mathrm{Graviton})^2$ anomaly conditions are not relevant to the constraints on the flavor charges of matter fields in the low-energy effective theory.

Because the charged matter fields consist of $(\mathbf{15},\ \mathbf{1})$, $(\mathbf{6}^*,\ \mathbf{2})$ and their conjugates under $SU(6) \times SU(2)_{\rm R}$, the mixed anomaly conditions become

$$\begin{aligned} 4 \left[\sum_{i=0}^{3} (a_i - q_\theta) + (\overline{a} - q_\theta) \right] & \\ + 2 \left[\sum_{i=0}^{3} (b_i - q_\theta) + (\overline{b} - q_\theta) \right] + 12\, q_\theta &\equiv 0, \qquad (\mathrm{mod}\ MN) \end{aligned} \tag{88}$$

$$6 \left[\sum_{i=0}^{3} (b_i - q_\theta) + (\overline{b} - q_\theta) \right] + 4\, q_\theta \equiv 0, \qquad (\mathrm{mod}\ MN) \tag{89}$$

for $SU(6)$ and $SU(2)_{\rm R}$, respectively. These conditions are rewritten as

$$4\, a_T + 2\, b_T \equiv 18\, q_\theta, \qquad 6\, b_T \equiv 26\, q_\theta, \qquad (\mathrm{mod}\ MN) \tag{90}$$

where

$$a_T = \sum_{i=0}^{3} a_i + \overline{a}, \qquad b_T = \sum_{i=0}^{3} b_i + \overline{b}. \tag{91}$$

Noting that a_T is even and b_T is odd, we obtain

$$a_T - b_T \equiv \frac{1}{2} MN - 2\, q_\theta, \qquad (\mathrm{mod}\ MN) \tag{92}$$

$$6\, a_T \equiv 14\, q_\theta. \qquad (\mathrm{mod}\ MN) \tag{93}$$

Because the Grassmann number θ has charge $(\pm 1,\ 0)$ under $\mathbf{Z}_M \times \mathbf{Z}_N$, in the case $M \equiv 0 \pmod 3$, we have no solutions of the anomaly condition Eq. (93). Thus

$$M \not\equiv 0. \qquad (\mathrm{mod}\ 3) \tag{94}$$

In a previous paper[1], we chose $M = 15$ and $N = 14$. This choice contradicts the above conditions. Therefore, in the next section we explore viable solutions that are consistent with these anomaly conditions. The anomaly conditions are so stringent that many types of discrete symmetries are ruled out. In fact, as seen in the next section, we find a LMA solution but no SMA solution.

Finally, we would like to remark that the $D_4 = \mathbf{Z}_2^{(\mathrm{F})} \ltimes \mathbf{Z}_4$ mixed anomaly conditions are satisfied in the present model. As seen from Tables II and III, under $\mathbf{Z}_2^{(\mathrm{FC})}$, $\phi(\mathbf{15},\ \mathbf{1})_i$ $(i = 0, 1, 2, 3)$ and $\overline{\phi(\mathbf{15},\ \mathbf{1})}$ are even, while $\psi(\mathbf{6}^*,\ \mathbf{2})_i$ $(i = 0, 1, 2, 3)$ and $\overline{\psi(\mathbf{6}^*,\ \mathbf{2})}$ are odd. Since these fields are even-dimensional representations of $SU(6)$ and also of $SU(2)_{\mathrm{R}}$, the present matter content is anomaly-free with respect to the $\mathbf{Z}_2^{(\mathrm{FC})}$ mixed anomaly. For the $\mathbf{Z}_4$ mixed anomalies, we have to take account of the relation

$$g_1\, g_2\, g_1^{-1} = g_2^{-1}. \tag{95}$$

Specifically, g_1 does not commutate with g_2 but does commutate with g_2^2. This relation implies that $\mathbf{Z}_4$ charges are additive not mod 4 but mod 2. Therefore, in order to determine whether the $\mathbf{Z}_4$ mixed anomaly conditions are satisfied, it is enough to determine whether $\mathbf{Z}_2^{(\mathrm{R})}$, which is a subgroup of $\mathbf{Z}_4$, is anomalous. As shown in Table IV, under $\mathbf{Z}_2^{(\mathrm{R})}$, $\phi(\mathbf{15},\ \mathbf{1})_i$ and $\psi(\mathbf{6}^*,\ \mathbf{2})_i$ $(i = 1, 2, 3)$ superfields are odd, while $\phi(\mathbf{15},\ \mathbf{1})_0$, $\psi(\mathbf{6}^*,\ \mathbf{2})_0$, $\overline{\phi(\mathbf{15},\ \mathbf{1})}$ and $\overline{\psi(\mathbf{6}^*,\ \mathbf{2})}$ are even. Their fermion components have opposite R-parities. Therefore, $\mathbf{Z}_2^{(\mathrm{R})}$ mixed anomalies of $\phi_0(\psi_0)$ and $\overline{\phi}(\overline{\psi})$ cancel pairwise with each other.

5 Anomaly-free solutions

In section 3 we studied a set of phenomenological conditions, which can be expressed as

$$-(b_0 + \overline{b}) = n \;\; = \;\; \frac{1}{4}(MN - q_\theta),$$

$$\begin{aligned}
-4\zeta_{00}+3a_0 &= 2q_\theta, \\
-4\eta_{00}+a_0+2b_0 &= -8n, \\
-4\mu_{33}+a_3+b_3+b_0 &= 2q_\theta, \\
-4\zeta_{33}+2a_3+a_0 &= 2q_\theta, \\
-4\eta_{33}+2b_3+a_0 &= 2q_\theta, \\
-4\nu_{33}+2b_3+2\overline{b} &= -4n+q_\theta.
\end{aligned} \tag{96}$$

Desirable values of the colored Higgs mass and μ are obtained in the case

$$\zeta_{00}\sim 0, \qquad \eta_{00}\sim 2n. \tag{97}$$

The observed fermion mass spectra require parameterizations in which $\mu_{33}\sim 0$, $x^{\alpha_1}\sim\lambda^3$, $\quad x^{\beta_1}\sim\lambda^4$ and $x^{\alpha_2}\sim x^{\beta_2}\sim\lambda^2$. In order to account for the observed pattern of the CKM matrix, we impose the condition

$$\zeta_{33}\sim\beta_1-\alpha_1+\mu_{33}+\frac{1}{2}\left(\frac{n}{4}-1\right). \tag{98}$$

The LMA solution is obtained under the condition

$$\eta_{33}\sim\frac{\alpha_1+\alpha_2}{2}-\beta_1+\mu_{33}+\frac{1}{2}\left(\frac{n}{4}-1\right), \tag{99}$$

while the condition for the SMA solution becomes

$$\eta_{33}\sim\alpha_1-\beta_2+\mu_{33}+\frac{1}{2}\left(\frac{n}{4}-1\right). \tag{100}$$

In addition, from Eq. (70) we have the condition

$$\nu_{33}\sim\frac{2}{3}n. \tag{101}$$

When M, N and q_θ are given, and when ζ_{00}, η_{00}, μ_{33}, ζ_{33}, η_{33} and ν_{33} are also given, we have too many relations, because there are five undetermined $\mathbf{Z}_{MN}$ charges a_0, b_0, $\overline{b}$, a_3 and b_3, with the seven equations given in Eq. (96). The existence of a solution is not certain, and proving or disproving its existence is a subtle matter.

As discussed in the previous section, the anomaly conditions are given by

$$a_T-b_T \equiv \frac{1}{2}MN-2q_\theta, \qquad (\text{mod } MN) \tag{102}$$

$$6a_T \equiv 14q_\theta. \qquad (\text{mod } MN) \tag{103}$$

¿From the parameterization represented by $a_0+\overline{a}=-4,\ b_0+\overline{b}=-n=-(MN-q_\theta)/4$ and Eq. (45), a_T and b_T can be rewritten as

$$a_T = 3a_3 + 4(\alpha_1+\alpha_2) - 4, \qquad b_T = 3b_3 + 4(\beta_1+\beta_2) - n. \tag{104}$$

Recalling that $x^{\alpha_1} \sim \lambda^3 \sim 10^{-2}$, and so forth, and that $x^{2n-1} \sim 2\times 10^{-17}$, we obtain the relations

$$\alpha_1+\alpha_2 \sim 0.4\times n, \qquad \beta_1+\beta_2 \sim 0.5\times n. \tag{105}$$

Solutions of Eqs. (102) and (103) are found only in the case

$$a_T - b_T = \frac{1}{2}MN - 2q_\theta, \tag{106}$$

$$a_T = \frac{1}{3}(7q_\theta + 2MN). \tag{107}$$

After some tedious calculations, we find a LMA solution for which

$$\begin{aligned} M = 19, \qquad N = q_\theta = 18, \qquad n = 81, \\ a_T = 270, \qquad b_T = 135 \end{aligned} \tag{108}$$

and $x^{161} \sim 2\times 10^{-17}$, $x^{6.3} = \lambda \simeq 0.22$. $\mathbf{Z}_{MN}$ charges ($MN = 342$) of the matter fields are listed in Table V. This parameterization leads to

$$(\zeta_{00},\ \eta_{00},\ \mu_{33},\ \zeta_{33},\ \eta_{33},\ \nu_{33}) = (0,\ 158,\ 3,\ 17,\ 2,\ 51). \tag{109}$$

The scales of the colored Higgs mass and μ are

$$m_{g_0/g_0^c} \simeq \langle S_0\rangle = x^{0.5}\times M_S \sim M_S, \tag{110}$$

$$\mu \simeq x^{154.5}\times M_S \sim 100 \text{ GeV}. \tag{111}$$

The quark/lepton mass spectra at the scale M_S become

$$\begin{aligned} (m_u,\ m_c,\ m_t) &\sim (\lambda^{7.8},\ \lambda^{5.2},\ \lambda^{0.5})\times v_u, \\ (m_d,\ m_s,\ m_b) &\sim (\lambda^{7.8},\ \lambda^{6.7},\ \lambda^{3.5})\times v_d, \\ (m_e,\ m_\mu,\ m_\tau) &\sim (\lambda^{7.8},\ \lambda^{5.9},\ \lambda^{2.7})\times v_d \end{aligned} \tag{112}$$

for $-\delta_d = \delta_L \sim 1$. These results are in accord with a small value of $\tan\beta \equiv v_u/v_d$. The CKM matrix turns out to be of the form

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^5 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{113}$$

Table 5: Assignment of $\mathbf{Z}_{342}$ charges for matter superfields

	Φ_1	Φ_2	Φ_3	Φ_0	$\overline{\Phi}$
$\phi(\mathbf{15},\ \mathbf{1})$	$a_1 = 126$	$a_2 = 102$	$a_3 = 46$	$a_0 = 12$	$\overline{a} = -16$
$\psi(\mathbf{6}^*,\ \mathbf{2})$	$b_1 = 120$	$b_2 = 80$	$b_3 = 16$	$b_0 = -14$	$\overline{b} = -67$

and the mixing angles in the MNS matrix become

$$\tan\theta_{12}^{\rm MNS} \sim \lambda^{0.7}, \qquad \tan\theta_{23}^{\rm MNS} \sim \lambda^{0.3}, \qquad \tan\theta_{13}^{\rm MNS} \sim \lambda. \tag{114}$$

The neutrino mass spectra are given by

$$(m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3}) \sim 10^{-1}{\rm eV} \times (\lambda^{1.9},\ \lambda^{0.6},\ 1). \tag{115}$$

Unlike the case for the LMA solution, we could not find phenomenologically viable SMA solutions in the parameter region $MN < 600$ and $m \equiv 0 \pmod 4$, because it is difficult to realize a situation in which the condition (101) is compatible with the other conditions. Recent experimental data on neutrino oscillations[24, 25, 26] strongly suggest that the LMA-MSW solution is most favorable. The result obtained here is consistent with these data.

6 Summary and discussion

In order to construct a string-inspired model that connects appropriately with low-energy physics, it is of great importance to explore both the gauge symmetry and the flavor symmetry at the string scale M_S. We chose $SU(6) \times SU(2)_{\rm R}$ as the unified gauge symmetry at M_S. The gauge symmetry can be derived from the perturbative heterotic superstring theory via the flux breaking. The symmetry breaking of $SU(6) \times SU(2)_{\rm R}$ down to $G_{\rm SM}$ can take place via the Higgs mechanism without matter fields of adjoint or higher representations. Because the doublet Higgs and the color-triplet Higgs fields exist in different irreducible representations, the triplet-doublet splitting problem is solved naturally. As the flavor symmetry, we introduced $\mathbf{Z}_M \times \mathbf{Z}_N$ and the dihedral group D_4 symmetries. $\mathbf{Z}_M$ and D_4 are R symmetries, while $\mathbf{Z}_N$ is a non-R symmetry. Introduction of the dihedral group D_4 is motivated by the phenomenological observation that the R-handed Majorana neutrino mass for the third generation is nearly equal to the geometrical average of M_S and M_Z. We assigned the appropriate flavor charges to the matter fields. After studying the mixed anomaly conditions, we solved them under many phenomenological constraints coming from the particle spectra. With the stringent anomaly conditions, a LMA-MSW solution was found, but no SMA-MSW solution was found. The solution includes phenomenologically acceptable results concerning fermion masses and mixings and also concerning hierarchical energy scales including the GUT scale, the μ scale and the Majorana mass scale of R-handed neutrinos.

We obtained the reasonable particle spectra at an energy scale around the scale M_S as shown in the previous section. In order to investigate the particle spectra at low-energies, we need to study the renormalization-group evolution of gauge couplings and the effective Yukawa couplings and to incorporate the supersymmetry breaking effect. In our LMA-MSW solution, the ratio m_d/m_e at M_S is nearly unity, and also we obtain $m_b/m_\tau \sim \lambda$ at M_S. These results are in contrast with those obtained from some conventional GUT-type models, in which the ratio m_b/m_τ is predicted to be unity at the GUT scale. In the present model, we have peculiar particle spectra. In particular, there appear colored superfields with even R-parity around the TeV region, which do not participate in proton decay. In the presence of these extra colored particles, the $SU(3)_c$ gauge coupling remains almost unchanged in the whole region ranging from M_Z to M_S. Therefore, the renormalization effects of $SU(3)_c$ in our model are expected to become rather large compared with those in conventional GUT-type models. Thus it seems that the particle spectra at M_S obtained here are consistent with those at low energies. A detailed study of the renormalization group evolution will be presented elsewhere.

In this paper we assumed that the flavor symmetry contains the semi-direct product group D_4, which is an extension of R-parity. It would be interesting to explore other possibilities for the semi-direct product flavor symmetry. Among them we may find more simple flavor symmetries, which could lead to phenomenologically viable results.

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